

**Math 131 - Quiz 5**

September 28, 2022

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 3.

1. (6 points) The following table gives information about the functions  $f$  and  $g$  and their derivatives at selected points.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	0	5	-1	-2
2	3	7	6	-1
3	5	4	9	2
4	4	-2	7	-3

- (a) Find  $h'(2)$  if  $h(x) = xg(x) - 2f(x)$ .

$$h'(x) = g(x) + xg'(x) - 2f'(x)$$

$$h'(2) = g(2) + 2g'(2) - 2f'(2) = 6 + 2(-1) - 2(7) = 6 - 2 - 14 = \boxed{-10}$$

- (b) Find  $h'(3)$  if  $h(x) = x^3g(x)$ .

$$h'(x) = 3x^2g(x) + x^3g'(x)$$

$$h'(3) = 3(3)^2g(3) + (3)^3g'(3) = 27(9 + 2) = \boxed{297}$$

- (c) Find  $h'(4)$  if  $h(x) = \frac{f(x)}{g(x)}$ .

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$h'(4) = \frac{g(4)f'(4) - f(4)g'(4)}{g(4)^2} = \frac{(7)(-2) - (4)(-3)}{7^2}$$

$$= \frac{-14 + 12}{49} = \boxed{-\frac{2}{49}}$$

Turn over.

2. (1 point) Use trig identities and the quotient rule to derive our formula for the derivative of  $y = \sec x$  from the derivative rule for cosine.

$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \tan x \quad \checkmark$$

3. (3 points) A potato is launched vertically upward with an initial velocity of 80 ft/s from a potato gun at the top of an 96-foot-tall building. The distance in feet that the potato travels from the ground after  $t$  seconds is given by  $s(t) = -16t^2 + 80t + 96$ .

- (a) Determine when the potato hits the ground.

$$-16t^2 + 80t + 96 = 0$$

$$-16(t^2 - 5t - 6) = 0$$

$$-16(t-6)(t+1) = 0 \Rightarrow t = 6 \text{ sec}$$

- (b) Determine the speed of the potato when it hits the ground.

$$s'(t) = -32t + 80$$

$$s'(6) = -32(6) + 80 = -112 \text{ FT/sec}$$

$$\text{SPEED} = 112 \text{ FT/sec}$$

- (c) Determine the maximum height of the potato.

$$s'(t) = 0 \Rightarrow -32t + 80 = 0$$

$$t = \frac{80}{32} = \frac{5}{2} = 2.5$$

$$s(2.5) = -16(2.5)^2 + 80(2.5) + 96$$

$$= 196 \text{ FT}$$