Math 131 - Quiz 6

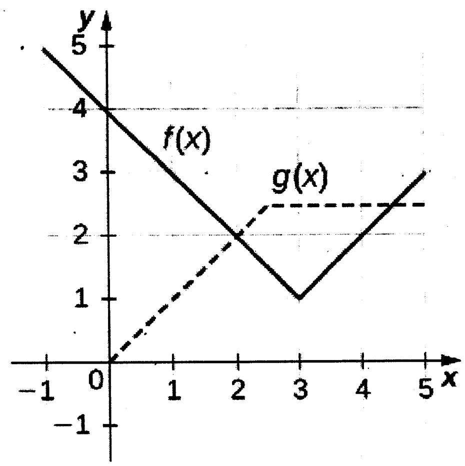
October 5, 2022

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 10.

1. (2 points) The graphs of f and g are shown below. Use the chain rule and information from the graphs to determine the derivative of $g(f(x))$ when $x = 1$.



$$\frac{d}{dx} g(f(x)) = g'(f(x))\, f'(x)$$

$$\left.\frac{d}{dx} g(f(x))\right|_{x=1} = g'(f(1))\, f'(1) = g'(3)\, f'(1)$$

$$= (0)(-1) = \boxed{0}$$

2. (1 point) Determine $f'(x)$ if $f(x) = \tan(\pi x^2 + x)$.

$$f'(x) = \sec^2(\pi x^2 + x)\left[\frac{d}{dx}(\pi x^2 + x)\right]$$

$$= (2\pi x + 1)\sec^2(\pi x^2 + x)$$

3. (1 point) Let $h(x) = [f(x)]^3$. Given the following information, compute $h'(1)$.

$f(0) = 4, \quad f'(0) = -2, \quad f(1) = 2, \quad f'(1) = 6, \quad f(2) = 5, \quad f'(2) = -9$

$$h'(x) = 3[f(x)]^2 f'(x)$$

$$h'(1) = 3[f(1)]^2 f'(1) = 3(2)^2(6) = 3 \cdot 4 \cdot 6 = \boxed{72}$$

Turn over.

4. (2 points) Find an equation of the line tangent to the graph of $y = \sqrt{x^2 - 12}$ at the point where $x = 4$.

Slope:

$$\frac{dy}{dx} = \frac{1}{2}(x^2-12)^{-1/2}(2x)$$

$$m = \left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2}(4)^{-1/2}(8) = 2$$

Point:

$x = 4 \Rightarrow y = \sqrt{4} = 2$

Tan line:

$y - 2 = 2(x-4)$

or

$y = 2x - 6$

5. (1 point) Let $h(x) = g(f(x))$. Given the following information, compute $h'(2)$.

$$f(2) = 3, \quad f'(2) = 8, \quad g'(0) = 0, \quad g'(2) = -5, \quad g'(3) = 9$$

$$h'(x) = g'(f(x))\, f'(x)$$

$$h'(2) = g'(f(2))\, f'(2) = g'(3)\, f'(2) = 9 \cdot 8 = \boxed{72}$$

6. (3 points) Given the equation $x^3 + 8xy + y^3 = 25x$, use implicit differentiation to determine $\frac{dy}{dx}$ at the point $(x, y) = (1, 2)$.

$$\frac{d}{dx}\left[x^3 + 8xy + y^3\right] = \frac{d}{dx}\left[25x\right]$$

$$3x^2 + 8y + 8x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 25$$

$$(8x + 3y^2)\frac{dy}{dx} = 25 - 3x^2 - 8y$$

$$\frac{dy}{dx} = \frac{25 - 3x^2 - 8y}{8x + 3y^2}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{25 - 3 - 16}{8 + 12} = \frac{6}{20} = \boxed{\frac{3}{10}}$$