Math 131 - Quiz 7

October 19, 2022

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 24.

1. (2 points) Find an equation of the line tangent to the graph of $y = x\tan^{-1}(x^2)$ at the point where $x = 1$.

SLOPE:

$$\frac{dy}{dx} = \tan^{-1}(x^2) + x\left(\frac{2x}{1+x^4}\right)$$

$$m = \left.\frac{dy}{dx}\right|_{x=1} = \tan^{-1}(1) + 1 = 1 + \frac{\pi}{4}$$

POINT: $x = 1$, $y = (1)\tan^{-1}(1) = \frac{\pi}{4}$

TAN. LINE:

$$y - \frac{\pi}{4} = \left(1 + \frac{\pi}{4}\right)(x-1)$$

OR

$$y = \left(1+\frac{\pi}{4}\right)x - 1$$

2. (2 points) Find the exact value of each of these.

(a) $\cos^{-1}(\frac{1}{2})$

$$\cos x = \frac{1}{2} \Rightarrow x = \boxed{\frac{\pi}{3}}$$

(b) $\sin^{-1}(-1)$

$$\sin x = -1 \Rightarrow x = \boxed{-\frac{\pi}{2}}$$

3. (2 points) Determine each derivative

(a) $\frac{d}{dt}\left(\sqrt{\pi}e^{-t^2}\right) = \sqrt{\pi}e^{-t^2}(-2t) = -2\sqrt{\pi}\,t\,e^{-t^2}$

(b) $\frac{d}{dx}\ln(\sec x + \tan x) = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$

$$= \boxed{\sec x}$$

Turn over.

4. (2 points) Use logarithmic differentiation to find dy/dx when $y = \dfrac{(x+1)^3(x+2)}{x^5(x+3)}$.

$$\ln y = \ln\left[\frac{(x+1)^3(x+2)}{x^5(x+3)}\right] = 3\ln(x+1) + \ln(x+2) - 5\ln x - \ln(x+3)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x+1} + \frac{1}{x+2} - \frac{5}{x} - \frac{1}{x+3}$$

$$\boxed{\frac{dy}{dx} = \left[\frac{(x+1)^3(x+2)}{x^5(x+3)}\right]\left(\frac{3}{x+1} + \frac{1}{x+2} - \frac{5}{x} - \frac{1}{x+3}\right)}$$

5. (2 points) Determine each derivative.

(a) $\dfrac{d}{dy}\left[5^{y^2} + (y^2)^5\right] = \boxed{5^{y^2}(\ln 5)(2y) + 10y^9}$

y^{10}

(b) $\dfrac{d}{dx}\log_3[(5x+1)^4] = \dfrac{d}{dx}\dfrac{4\ln(5x+1)}{\ln 3} = \dfrac{4}{\ln 3}\dfrac{d}{dx}\ln(5x+1)$

$$= \boxed{\frac{4}{\ln 3}\cdot\frac{5}{5x+1}}$$