Math 131 - Test 2
October 12, 2022

Name Key

Score ________

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives and do not simplify.

1. (10 points) Let $f(x) = \sqrt{x}$. Use a **limit definition of the derivative** to determine $f'(x)$. Show all work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

2. (4 points) Which one of the following best describes the line tangent to the graph of $g(x) = |2x - 6|$ at the point $(3, 0)$? (Briefly explain or show work to receive full credit.)

   (a) The tangent line is horizontal.
   (b) The tangent line is vertical.
   (c) A unique tangent line does not exist.
   (d) The tangent line has slope 1.

$$g(x) = \begin{cases} 2x-6, & x \geq 3 \\ 6-2x, & x < 3 \end{cases}$$

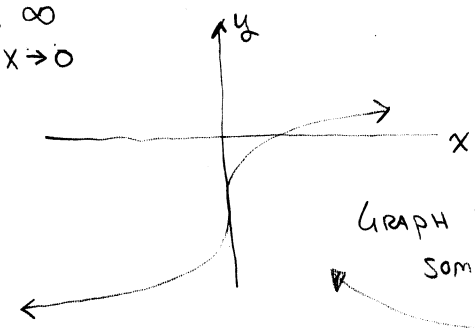
SLOPE FROM RIGHT OF $X = 3$ IS 2

SLOPE FROM LEFT OF $X = 3$ IS -2

3. (4 points) Which one of the following best describes the line tangent to the graph of $f(x) = 5x^{1/3} - 2$ at the point $(0, -2)$? (Briefly explain or show work to receive full credit.)

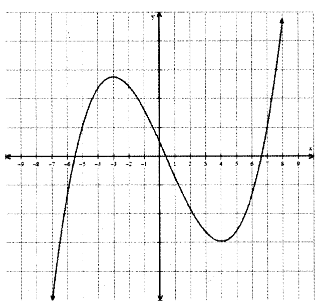
$f'(x) = \frac{5}{3} x^{-2/3} \leftarrow$ APPROACHES ∞ AS $X \to 0$

   (a) The tangent line is horizontal.
   (b) The tangent line is vertical.
   (c) A unique tangent line does not exist.
   (d) The tangent line has slope -2.



GRAPH LOOKS SOMETHING LIKE

4. (6 points) The graph of $g(x)$ is shown below. Choose the lettered graph that best represents the graph of $g'(x)$. Explain your reasoning. Give at least two reasons to support your answer.



(1) $g'(x) = 0$ AT $X = -3$ AND $X = 4$

MATCHES B.

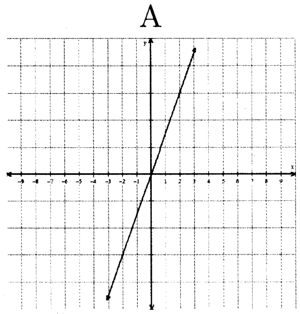
(2) TANGENT LINES HAVE NEGATIVE SLOPE BETWEEN -3 & 4.

MATCHES B.

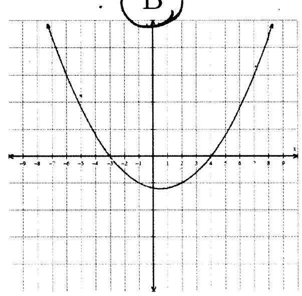
(3) TANGENT LINES HAVE POSITIVE SLOPE OUTSIDE OF $(-3, 4)$.

MATCHES B.

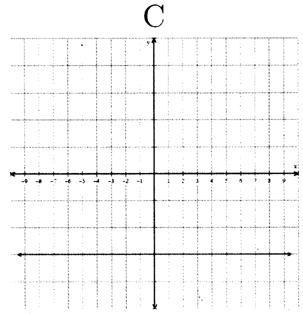
A



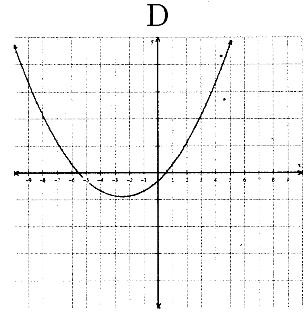
B



C



D



5. An object is launched vertically so that its height (in feet) after t seconds is given by

$$s(t) = -16t^2 + 72t + 40.$$

Include units with your answer for each part of this problem.

(a) (3 points) Determine the average rate of change the object's height over the interval from $t = 0$ to $t = 2$.

$$\frac{\Delta s}{\Delta t} = \frac{s(2) - s(0)}{2} = \frac{120 - 40}{2} = \boxed{40 \text{ FT/SEC}}$$

(b) (3 points) Determine the object's velocity at time $t = 3$.

$$v(t) = s'(t) = -32t + 72$$

$$s'(3) = \boxed{-24 \text{ FT/SEC}}$$

(c) (2 points) What is the acceleration of the object?

$$a(t) = s''(t) = \boxed{-32 \text{ FT/SEC}^2}$$

(d) (4 points) Determine the object's maximum height.

$$v(t) = 0 \Rightarrow -32t + 72 = 0 \Rightarrow t = 2.25 \text{ SEC}$$

$$s(2.25) = \boxed{121 \text{ FT}}$$

(e) (3 points) When does the object hit the ground?

$$-16t^2 + 72t + 40$$
$$= -8(2t^2 - 9t - 5) = -8(2t+1)(t-5) = 0 \Rightarrow \boxed{t = 5 \text{ SEC}}$$

(f) (1 point) What is the object's initial speed?

$$v(0) = \boxed{72 \text{ FT/SEC}}$$

(g) (1 point) What is the object's speed when it hits the ground?

FROM (e), HITS GROUND AT $t = 5$

$$\text{SPEED} = |v(5)| = |-88| = \boxed{88 \text{ FT/SEC}}$$

6. (20 points) Differentiate. Do not simplify.

(a) $\frac{d}{dt}\left(10+\frac{2}{t}-\frac{1}{t^2}+\sqrt[5]{t^3}\right) = \frac{d}{dt}\left(10+2t^{-1}-t^{-2}+t^{3/5}\right)$

SUM/DIFF & POWER RULES

$$= \boxed{0-2t^{-2}+2t^{-3}+\frac{3}{5}t^{-2/5}}$$

(b) $\frac{d}{dx}\left(\frac{4x^5-7x^2}{\csc x}\right) = \boxed{\frac{(\csc x)(20x^4-14x)-(4x^5-7x^2)(-\csc x\cot x)}{\csc^2 x}}$

QUOTIENT RULE

(c) $\frac{d}{dx}\sec(\sqrt{x}) = \sec\sqrt{x}\cdot\tan\sqrt{x}\cdot\frac{d}{dx}\sqrt{x}$

CHAIN RULE

$$= \boxed{\frac{1}{2}x^{-1/2}\sec\sqrt{x}\tan\sqrt{x}}$$

(d) $\frac{d}{dt}\left[(8t-3)^5\sin(t^2)\right] = \boxed{5(8t-3)^4(8)\sin(t^2)+(8t-3)^5\cos(t^2)(2t)}$

PRODUCT & CHAIN RULES

7. (6 points) Let $f(x) = 8x\cos x$. Find $f''(x)$.

$f'(x) = 8\cos x - 8x\sin x$

$f''(x) = -8\sin x - 8\sin x - 8x\cos x$

$$= \boxed{-16\sin x - 8x\cos x}$$

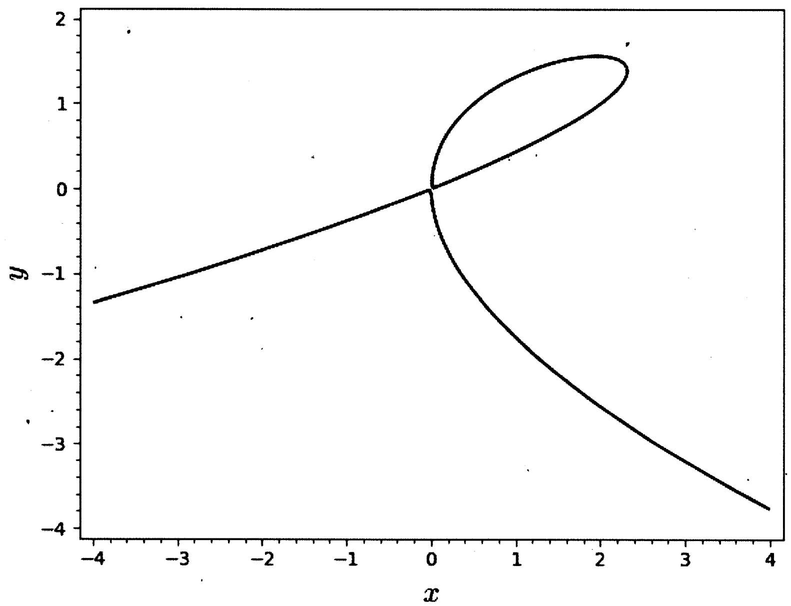
8. (4 points) The line $y = 2x + 3$ is tangent to a graph at the point $(3, 9)$. Find an equation for the line normal to the graph at $(3, 9)$.

Slope of normal line is $-\frac{1}{2}$

Normal line:

$$\boxed{y - 9 = -\frac{1}{2}(x-3) \quad \text{or} \quad y = -\frac{1}{2}x + \frac{21}{2}}$$

9. (12 points) The graph of the equation $x^2 + y^3 = \frac{5}{2}xy$ is shown below.



(a) Use implicit differentiation to find a formula for dy/dx.

$$\frac{d}{dx}(x^2 + y^3) = \frac{d}{dx}\left(\frac{5}{2}xy\right)$$

$$2x + 3y^2 \frac{dy}{dx} = \frac{5}{2}y + \frac{5}{2}x\frac{dy}{dx}$$

$$\left(3y^2 - \frac{5}{2}x\right)\frac{dy}{dx} = \frac{5}{2}y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{\frac{5}{2}y - 2x}{3y^2 - \frac{5}{2}x}}$$

Typo!

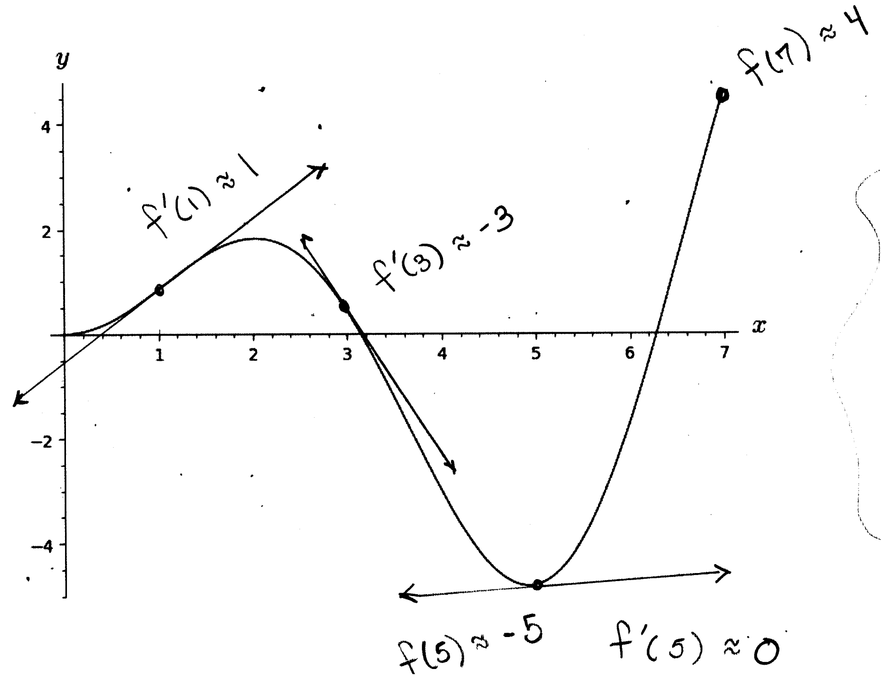
(b) Use dy/dx to compute the slope of the graph at the point $(1, 2)$. Then determine an equation for the tangent line at $(2, 1)$.

$$m = \left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{\frac{5}{2} - 4}{3 - 5} = \frac{-\frac{3}{2}}{-2} = \frac{3}{4}$$

$$\boxed{y - 1 = \frac{3}{4}(x-2) \quad \text{or} \quad y = \frac{3}{4}x - \frac{1}{2}}$$

10. (8 points) The graph of the function f is shown below. Referring to the graph, place the following values in order from least to greatest. Explain or show work to receive full credit.

$$f'(1), \quad f'(5), \quad f(5), \quad f(7), \quad f'(3)$$



Least to greatest:

$f(5)$, $f'(3)$, $f'(5)$, $f'(1)$, $f(7)$.

11. (6 points) Let $f(x) = x^5 + x^3 - 30$. Compute $(f^{-1})'(10)$.

$$(f^{-1})'(10) = \frac{1}{f'(f^{-1}(10))} = \frac{1}{f'(2)} = \boxed{\frac{1}{92}}$$

$f^{-1}(10) = 2$

$f'(x) = 5x^4 + 3x^2$

$f'(2) = 92$

$f^{-1}(10) = x \Rightarrow f(x) = 10$

$x^5 + x^3 - 30 = 10 \Rightarrow x = 2$ (Guess & Check!)

12. (3 points) Let $g(x) = 7x + 5$. Determine $(g^{-1})'(x)$.

$$(g^{-1})'(x) = \frac{1}{g'(g^{-1}(x))} = \boxed{\frac{1}{7}}$$

$g'(x) = 7$