Quiz 3

ⓘ This is a preview of the published version of the quiz

Started: Sep 18 at 3:33pm

Quiz Instructions

Choose the best solution choice for each multiple-choice problem. The last two problems require you to submit written work. The problems vary in value from 1 to 2 points.

Question 1 — 2 pts

Determine the value of k so that g is continuous everywhere.

$$g(x) = \begin{cases} 3x^2 - kx + 5, & x < 2 \\ x^2 + \sin(\pi x), & x \geq 2 \end{cases}$$

$g(2) = (2)^2 + \sin(2\pi) = 4 + 0$

$= 4 = \lim_{x \to 2^+} g(x)$

- ○ $-5/2$
- ○ k can be any real number.
- ☒ $13/2$
- ○ There is no value of k that will make the function continuous.

$\lim_{x \to 2^-} g(x) = 3(2)^2 - k(2) + 5 = 17 - 2k$

$17 - 2k = 4$

$\Rightarrow k = \frac{13}{2}$

Question 2 — 2 pts

The function f has two discontinuities. Find and classify them.

$$f(x) = \begin{cases} x^2/|x-1|, & x < 1 \\ 15/(x^2-1), & 1 < x \leq 4 \\ x^2 - 14, & x > 4 \end{cases}$$

- ○ $x = 1$ (Jump) and $x = 4$ (Jump)
- ○ $x = 1$ (Infinite) and $x = 4$ (Removable)

THE PIECES ARE INDIVIDUALLY CONTINUOUS EVERYWHERE, SO THE ONLY POSSIBLE DISCONTS. ARE WHERE THE PIECES GO FROM ONE TO THE NEXT. THEREFORE THE POSSIBLE DISCONTS ARE $X = 1$ AND $X = 4$. LET'S CHECK THEM. NEXT PAGE →

X = 1 IS AN INFINITE DISCONT. BOTH ONE-SIDED LIMITS AT X = 1 GIVE NONZERO/ZERO FORMS.

- ○ $x = 1$ (Jump) and $x = 4$ (Removable)
- ☒ $x = 1$ (Infinite) and $x = 4$ (Jump)

X = 4 IS A JUMP DISCONT BECAUSE $\lim_{x \to 4^-} f(x) = 1 \neq \lim_{x \to 4^+} f(x) = 2$.

Question 3

1 pts

Go back and read the informal definition of limit from the **lecture 3 notes** **(http://stevekifowit.com/archives/M131/lect3.pdf)**. Compare that definition to the formal definition in the **lecture 9 notes** **(http://stevekifowit.com/archives/M131/lect9.pdf)**. Which quantity in the formal definition is used to quantify the arbitrary closeness of $f(x)$ and L?

- ☒ ϵ
- ○ c
- ○ L
- ○ δ

SEE THE NOTES.

Question 4

1 pts

Go back and read the informal definition of limit from the **lecture 3 notes** **(http://stevekifowit.com/archives/M131/lect3.pdf)**. Compare that definition to the formal definition in the **lecture 9 notes** **(http://stevekifowit.com/archives/M131/lect9.pdf)**. Which quantity in the formal definition is used to quantify the sufficient closeness of x and c?

- ○ c
- ○ L
- ☒ δ
- ○ ϵ

SEE THE NOTES.

Question 5

2 pts

Solve the following problem on paper, showing all work. Then submit your work as a pdf, jpg, or png file.

Find an interval of length one that contains a solution of the equation $x^5 - 3x^3 + 7x - 13 = 0$. Use the Intermediate Value Theorem to explain and justify your answer.

SEE ATTACHED SHEET.

Upload Choose a File

Question 6

2 pts

Solve the following problem on paper, showing all work. Then submit your work as a pdf, jpg, or png file.

Find and classify the discontinuities of $r(x) = \dfrac{x^2 + 6x - 7}{x^2 - 1}$.

SEE ATTACHED SHEET.

Upload Choose a File

Not saved Submit Quiz

Quiz 3 – Question 5

Let $f(x) = x^5 - 3x^3 + 7x - 13$.

f is continuous for all values of x.

Notice that $f(1) = (1)^5 - 3(1)^3 + 7(1) - 13$

$= 1 - 3 + 7 - 13 = -8.$

Also notice that $f(2) = (2)^5 - 3(2)^3 + 7(2) - 13$

$= 32 - 24 + 14 - 13 = 9$

Since $f(1) = -8$ and $f(2) = 9$,

f must take on any y-value between -8 and 9 on the interval where $1 \leq x \leq 2$.

In particular, $f(x)$ must be zero somewhere between $x = 1$ and $x = 2$.

Quiz 3 – Question 6

$$r(x) = \frac{x^2 + 6x - 7}{x^2 - 1} = \frac{(x+7)(x-1)}{(x+1)(x-1)}$$

There are two discontinuities arising from a zero denominator: $x = -1$, $x = 1$

$x = -1$

$$\lim_{x \to -1} \frac{(x+7)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \underbrace{\lim_{x \to -1} \frac{(x+7)}{(x+1)}}$$

This gives a $\frac{6}{0}$ form.

This is enough to know that $x = -1$ is a nonremovable, infinite discont.

$x = 1$

$$\lim_{x \to 1} \frac{(x+7)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{8}{2} = 4$$

Limit exists!

$x = 1$ is a removable discont.