Test 2

ⓘ This is a preview of the published version of the quiz

Started: Oct 23 at 10:40am

Quiz Instructions

Choose the best solution choice for each multiple-choice problem. For the problems that require an exact numerical answer, the answer will always be an integer. For problems that require written work, show all work to receive full credit. Submit your written work in one of the formats: pdf, jpg, or png.

The test is worth a total of 100 points. Point values are indicated on each problem.

There is no time limit, but you get only one attempt. Please note that the multiple-choice problems are worth several points each. **Be sure of your answers before submission.** Good luck!

Question 1 — 15 pts

Do all parts of this problem on paper, showing all work. Then submit your solutions as a pdf, jpg, or png file.

Use derivative rules to determine each derivative. Do not simplify.

A. $\dfrac{d}{dt}\left(10 + \dfrac{2}{t} - \dfrac{1}{t^2} + \sqrt[5]{t^3}\right)$

B. $\dfrac{d}{dx}\sec(\sqrt{x})$

SEE ATTACHED SHEET.

C. $\dfrac{d}{dt}\left[(8t-3)^5 \sin(t^2)\right]$

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Question 2 — 4 pts

Let $h(x) = \dfrac{f(x)}{\cos x}$. Given the following information, compute $h'(0)$.

$f(1) = 7, \quad f'(1) = -4, \quad f(0) = 8, \quad f'(0) = 12$

$$h'(x) = \frac{(\cos x) f'(x) + f(x) \sin x}{\cos^2 x}$$

Write your exact numerical answer in the box below.

12

$$h'(0) = \frac{(1) f'(0) + f(0)(0)}{(1)^2} = f'(0) = \boxed{12}$$

Question 3

4 pts

QUOTIENT RULE

Determine dy/dx when $y = \dfrac{4x^5 - 7x^2}{\csc x}$.

$$\frac{dy}{dx} = \frac{(\csc x)(20x^4 - 14x) - (4x^5 - 7x^2)(-\csc x \cot x)}{\csc^2 x}$$

- X $\dfrac{dy}{dx} = \dfrac{(20x^4 - 14x)\csc x + (4x^5 - 7x^2)\csc x \cot x}{\csc^2 x}$
- ○ $\dfrac{dy}{dx} = x(x(-7 + 4x^3)\cos(x) - 2(-7 + 10x^3)\sin(x))$
- ○ $\dfrac{dy}{dx} = \dfrac{(20x^4 - 14x)\csc x - (4x^5 - 7x^2)\sin x}{\csc^2 x}$
- ○ $\dfrac{dy}{dx} = -\dfrac{20x^4 - 14x}{\csc x \cot x}$

Question 4

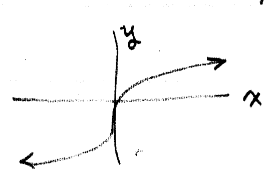
3 pts

Which one of the following best describes the line tangent to the graph of $f(x) = 5x^{1/3} - 2$ at the point $(0, -2)$?

$$f'(x) = \frac{5}{3} x^{-2/3} = \frac{5}{3x^{2/3}}$$

- ○ The tangent line is horizontal.

GRAPH OF f:

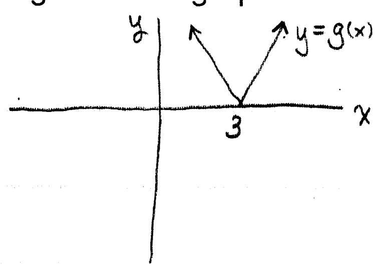


- A unique tangent line does not exist.
- The tangent line is vertical.
- The tangent line has slope -2.

Question 5

3 pts

Which one of the following best describes the line tangent to the graph of $g(x) = |2x - 6|$ at the point $(3, 0)$?



- The tangent line is vertical.
- The tangent line has slope 2.
- A unique tangent line does not exist.
- The tangent line is horizontal.

Question 6

4 pts

Let $f(x) = 8x\ \cos x$. Find the second derivative, $f''(x)$.

- $f''(x) = -8(x\cos(x) - 2\sin(x))$
- $f''(x) = -16\sin x - 8x\ \cos x$
- $f''(x) = 8\cos x - 8x\ \sin x$
- $f''(x) = -16x\ \cos x + 8x\ \sin x$

$f'(x) = 8\cos x - 8x\sin x$

$f''(x) = -8\sin x - 8\sin x - 8x\cos x$

Question 7

20 pts

Do all parts of this problem on paper, showing all work. Then submit your solutions as a pdf, jpg, or png file.

An object is launched vertically so that its height (in feet) after t seconds is given by

$s(t) = -16t^2 + 72t + 40.$

Include units with your answer for each part of this problem.

A. Determine the average rate of change of the object's height over the interval from $t = 0$ to $t = 2$.
B. Determine the object's velocity at time $t = 3$.
C. What is the acceleration of the object?
D. Determine the object's maximum height.
E. When does the object hit the ground?
F. What is the object's initial speed?
G. What is the object's speed when it hits the ground?

SEE ATTACHED SHEET.

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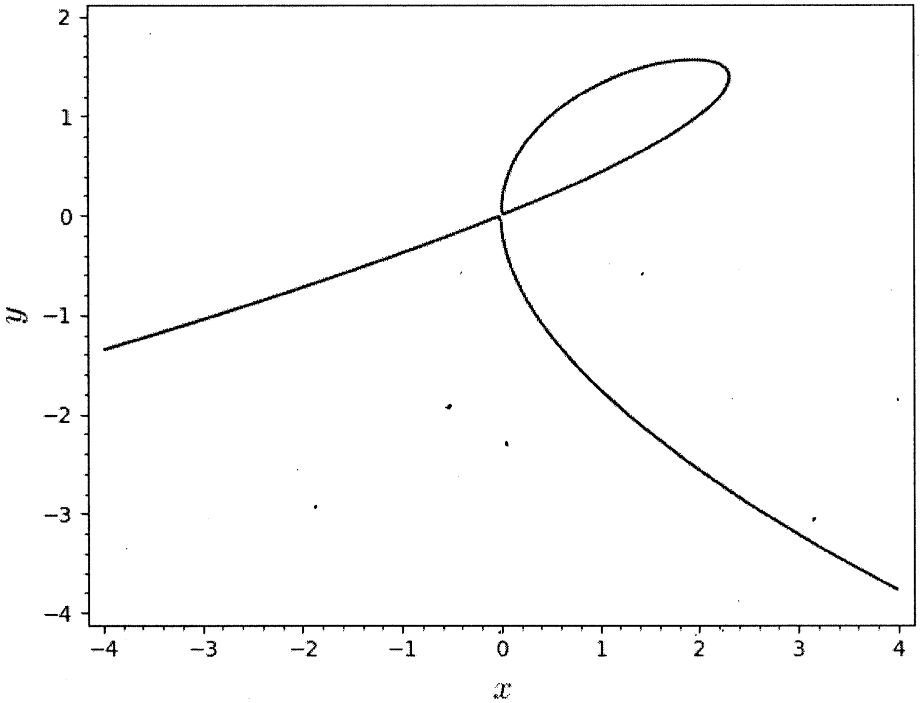
Question 8

12 pts

Do all parts of this problem on paper, showing all work. Then submit your solutions as a pdf, jpg, or png file.

The graph of the equation $x^2 + y^3 = \frac{5}{2}xy$ is shown below.

SEE ATTACHED SHEET.



A. Use implicit differentiation (the techniques from the lecture 16 notes (http://stevekifowit.com/archives/M131/lect16.pdf)) to find a formula for dy/dx.
B. Use dy/dx to find an equation of the line tangent to the graph at the point $(2, 1)$.

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Question 9

4 pts

A particle is moving along the x-axis in such a way that its position at any time t is given by $s(t) = -2t^3 + 12t^2 + 5t$. At what time is the particle's acceleration equal to zero?

Enter your exact numerical answer in the box below.

2

$S'(t) = -6t^2 + 24t$

$S''(t) = -12t + 24$

$S''(t) = 0 \Rightarrow t = \frac{24}{12} = 2$

Question 10

3 pts

NORMAL LINE IS PERPENDICULAR TO TANGENT LINE.

Suppose that the line $y = 2x + 1$ is tangent to a graph at the point $(4, 9)$. Find an equation for the line normal to the graph at $(4, 9)$.

NORMAL LINE HAS SLOPE $-\frac{1}{2}$

- ○ $y = -\frac{1}{2}x + 9$
- ○ $y = \frac{1}{2}x + 7$
- X $y = -\frac{1}{2}x + 11$
- ○ $y = -2x + 17$

$y - 9 = -\frac{1}{2}(x-4)$

$\boxed{y = -\frac{1}{2} + 11}$

Question 11

5 pts

Let $f(x) = x^5 + x^3 - 30$. Find $f^{-1}(10)$ and then find $(f^{-1})'(10)$. $= \dfrac{1}{f'(f^{-1}(10))}$

- X $f^{-1}(10) = 2$ and $(f^{-1})'(10) = \frac{1}{92}$
- ○ $f^{-1}(10) = 100970$ and $(f^{-1})'(10) = \frac{1}{50300}$
- ○ $f^{-1}(10) = 30$ and $(f^{-1})'(10) = 50300$
- ○ $f^{-1}(10) = 2$ and $(f^{-1})'(10) = \frac{1}{40}$

$f^{-1}(10) = x \Leftrightarrow f(x) = 10$

$x^5 + x^3 - 30 = 10$

GUESS & CHECK $\Rightarrow \boxed{x = 2}$

$f'(x) = 5x^4 + 3x^2$

$\dfrac{1}{f'(f^{-1}(10))} = \dfrac{1}{f'(2)} = \dfrac{1}{80+12} = \boxed{\dfrac{1}{92}}$

Question 12

3 pts

Let $g(x) = 7x + 5$. Determine $(g^{-1})'(x)$, the derivative of the inverse function.

○

$(g^{-1})'(x) = \dfrac{1}{g'(g^{-1}(x))} = \dfrac{1}{7}$

$g'(x) = 7$

$(g^{-1})'(x) = \dfrac{-7}{(7x+5)^2}$

$\cancel{\bigcirc}$ $(g^{-1})'(x) = \dfrac{1}{7}$

○ $(g^{-1})'(x) = \dfrac{1}{7x+5}$

○ $(g^{-1})'(x) = 7$

Question 13

4 pts

Let $h(x) = \sin^{-1}(f(x))$. Given the following information, compute $h'(3)$.

$$f(1) = \frac{1}{3}, \quad f'(1) = \frac{\sqrt{5}}{2}, \quad f(3) = \frac{\sqrt{3}}{2}, \quad f'(3) = \frac{1}{2}$$

$h'(x) = \dfrac{1}{\sqrt{1-f(x)^2}} \cdot f'(x)$

Enter your exact numerical answer in the box below.

1

$h'(3) = \dfrac{f'(3)}{\sqrt{1-f(3)^2}} = \dfrac{1/2}{\sqrt{1-3/4}} = \dfrac{1/2}{1/2} = 1$

Question 14

4 pts

Find $\dfrac{dy}{dx}$ if $y = 5\tan^{-1}(x^2)$.

$\dfrac{dy}{dx} = \dfrac{5}{1+(x^2)^2} \cdot 2x = \boxed{\dfrac{10x}{1+x^4}}$

○ $\dfrac{dy}{dx} = \dfrac{5}{1+x^2}$

○ $\dfrac{dy}{dx} = \dfrac{10x}{1+x^2}$

○ $\dfrac{dy}{dx} = \dfrac{10x}{\sec^2(x^2)}$

$\frac{dy}{dx} = \frac{10x}{1+x^4}$

Question 15

4 pts

Find the instantaneous rate of change of $f(x) = e^{5x^2-3x-2}$ at the point where $x = 1$

.

Enter your exact numerical answer in the box below.

7

$f'(x) = e^{5x^2-3x-2}(10x-3)$

$f'(1) = e^0(10-3) = \boxed{7}$

Question 16

4 pts

Find the slope of the line tangent to the graph of $y = \log_2(x^3 + 1)$ at the point where $x = 2$. Round your answer to the nearest hundredth.

- 1.92
- 1.60
- 1.33
- 0.92

$y = \frac{\ln(x^3+1)}{\ln 2}$ $\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{3x^2}{x^3+1}$

$\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{\ln 2} \cdot \frac{12}{9} \approx 1.923593$

Question 17

4 pts

Use logarithmic differentiation to compute $\frac{dy}{dx}$ when $y = \frac{x^3(x-5)}{3x+7}$.

Which of these is a correct step in the process?

$\cancel{\bigcirc}$ $\ln y = 3\ln(x) + \ln(x-5) - \ln(3x+7)$

○ $\frac{dy}{dx} = \frac{3}{x} + \frac{1}{x-5} + \frac{3}{3x+7}$

○ $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^3} + \frac{1}{x-5} - \frac{1}{3x+7}$

○ $\frac{1}{y}\frac{dy}{dx} = x^3 + (x-5) - (3x+7)$

① Log of both sides:

$$\ln y = \ln\left(\frac{x^3(x-5)}{3x+7}\right)$$

② Expand:

$$\ln y = 3\ln x + \ln(x-5) - \ln(3x+7)$$

③ $$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{1}{x-5} - \frac{3}{3x+7}$$

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④ $$\frac{dy}{dx} = \left(\frac{x^3(x-5)}{3x+7}\right)\left(\frac{3}{x} + \frac{1}{x-5} - \frac{3}{3x+7}\right)$$

#1

A) $\frac{d}{dt}\left(10 + \frac{2}{t} - \frac{1}{t^2} + \sqrt[5]{t^3}\right)$

$$= \frac{d}{dt}\left(10 + 2t^{-1} - t^{-2} + t^{3/5}\right) = \boxed{0 - 2t^{-2} + 2t^{-3} + \frac{3}{5}t^{-2/5}}$$

B) $\frac{d}{dx} \sec(\sqrt{x})$

$$= \boxed{\sec(\sqrt{x})\tan(\sqrt{x})\left(\frac{1}{2}x^{-1/2}\right)}$$

C) $\frac{d}{dt}\left[(8t-3)^5 \cdot \sin(t^2)\right] = \frac{d}{dt}\left[(8t-3)^5\right] \cdot \sin(t^2)$

$$+ (8t-3)^5 \cdot \frac{d}{dt}\sin(t^2)$$

$$= \boxed{5\cdot(8t-3)^4 \cdot 8 \cdot \sin(t^2) + (8t-3)^5 \cdot \cos(t^2) \cdot (2t)}$$

#7 $s(t) = -16t^2 + 72t + 40$

A) $\frac{\Delta s}{\Delta t} = \frac{s(2)-s(0)}{2-0} = \frac{120-40}{2} = \boxed{40 \text{ FT/SEC}}$

B) $v(t) = s'(t) = -32t + 72$. $s'(3) = -96 + 72 = \boxed{-24 \text{ FT/SEC}}$

C) $a(t) = s''(t) = \boxed{-32 \text{ FT/SEC}^2}$

D) $v(t) = 0 \Rightarrow -32t + 72 = 0 \Rightarrow t = \frac{72}{32} = 2.25 \text{ SEC}$

$S(2.25) = \boxed{121 \text{ FT}}$

E) $s(t) = 0 \Rightarrow -16t^2 + 72t + 40$

$= -8(2t^2 - 9t - 5) = -8(2t+1)(t-5) = 0$

$\Rightarrow \boxed{t = 5 \text{ SEC}}$

F) $v(0) = \boxed{72 \text{ FT/SEC}}$

G) HITS GROUND AT $t = 5$. $|v(5)| = |-88| = \boxed{88 \text{ FT/SEC}}$

#8 $x^2 + y^3 = \frac{5}{2}xy$

A) $\frac{d}{dx}\left(x^2 + y^3\right) = \frac{d}{dx}\left(\frac{5}{2}xy\right)$

$$2x + 3y^2\frac{dy}{dx} = \frac{5}{2}y + \frac{5}{2}x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - \frac{5}{2}x\frac{dy}{dx} = \frac{5}{2}y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{\frac{5}{2}y - 2x}{3y^2 - \frac{5}{2}x}}$$

B) $m = \left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{\frac{5}{2} - 4}{3 - 5} = \frac{-3/2}{-2} = \frac{3}{4}$

TANGENT LINE AT $(2,1)$:

$$\boxed{y - 1 = \frac{3}{4}(x-2) \quad \text{OR} \quad y = \frac{3}{4}x - \frac{1}{2}}$$