Test 3

ⓘ This is a preview of the published version of the quiz

Started: Nov 21 at 2:36pm

Quiz Instructions

Choose the best solution choice for each multiple-choice problem. For the problems that require an exact numerical answer, the answer will always be an integer. For problems that require written work, show all work to receive full credit. Submit your written work in one of the formats: pdf, jpg, or png.

The test is worth a total of 100 points. Point values are indicated on each problem.

There is no time limit, but you get only one attempt. Please note that the multiple-choice problems are worth several points each. **Be sure of your answers before submission.** Good luck!

Question 1 **4 pts**

Use the given information to find the linearization of $g(x)$ at $x = 3$. Use $L(x)$ to represent the linearization.

$g(2) = 4.21, \quad g'(2) = 0.44, \quad g(3) = 4.90, \quad g'(3) = 1.67$

○ $L(x) = 1.055x + 4.555$

○ $L(x) = 1.67x + 4.90$

○ $L(x) = 0.44x + 2.89$

✱ $L(x) = 1.67x - 0.11$

$L(x) = g(3) + g'(3)(x-3)$

$L(x) = 4.90 + 1.67(x-3)$

$\boxed{L(x) = 1.67x - 0.11}$

Question 2 **6 pts**

Do this problem on paper, showing all work. Then submit your solution as a pdf, jpg, or png file.

Determine the differential dy when $y = e^{5x} \cos x$.

SEE ATTACHED SHEET.

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Question 3

4 pts

Find the linearization of $f(x) = \tan^{-1} x$ at $x = 1$. Then use your linearization to approximate $f(0.92)$.

$f'(x) = \frac{1}{1+x^2}$

$f(1) = \tan^{-1}(1) = \frac{\pi}{4}$, $f'(1) = \frac{1}{2}$

$L(x) = \frac{\pi}{4} + \frac{1}{2}(x-1)$

$L(0.92) = \frac{\pi}{4} + \frac{1}{2}(-0.08) = 0.745398$

- ○ $f(0.92) \approx 0.7551$
- ○ $f(0.92) \approx 0.7438$
- ✱ $f(0.92) \approx 0.7454$
- ○ $f(0.92) \approx 0.7853$

Question 4

2 pts

Explain why $x = 0$ is NOT a critical number of the function $f(x) = \sqrt{x}$.

f IS DEFINED ON $[0, \infty)$

- ○ $f(0)$ is not defined.
- ○ $f'(0)$ is not defined.
- ○ $x = 0$ is not in the domain of f.
- ✱ $x = 0$ is a domain endpoint.

Question 5

10 pts

Do this problem on paper, showing all work. Then submit your solution as a pdf, jpg, or png file.

Determine the critical numbers of $g(x) = 3x - 10x^{3/5}$.

SEE ATTACHED SHEET.

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Question 6

4 pts

Suppose you were to use calculus techniques (i.e., those from the **lecture 22 notes (http://stevekifowit.com/archives/M131/lect22.pdf)**) to find the absolute extreme values of $g(x) = 8x - x^4$ on the interval $[-2, 1]$. Which of these function values would be required in order for you to draw your conclusions?

CRIT #'S: $g'(x) = 8 - 4x^3 = 0$

$\Rightarrow x^3 = 2$

$\Rightarrow x = \cancel{\sqrt[3]{2} \approx 1.26}$

END POINTS: $x = -2,\ x = 1$

$x \approx 1.26 > 1$. IT IS OUTSIDE THE DOMAIN.

- ✱ $g(-2)$, $g(1)$, and no other values
- ○ $g(-2)$, $g(\sqrt[3]{2})$, $g(1)$, and no other values
- ○ $g(\sqrt[3]{2})$ and no other values
- ○ $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, and no other values

Question 7

16 pts

Do this problem on paper, showing all work. Then submit your solution as a pdf, jpg, or png file.

Let $f(x) = x^5 - 10x^4 + 25x^3$. Apply the 1st and 2nd derivative tests to find open intervals on which f is increasing/decreasing and open intervals on which the graph of f is concave up/down. Also identify all relative extreme values and inflection points. (You will find it useful to know that the solutions of $2x^2 - 12x + 15 = 0$ are $x = \frac{6-\sqrt{6}}{2} \approx 1.775$ and $x = \frac{6+\sqrt{6}}{2} \approx 4.225$.)

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Question 8 6 pts

The function f has critical numbers $x = -3$, $x = 0$, and $x = 2$. Use the information given below to determine whether each critical number gives a relative minimum, a relative maximum, or neither.

2ND DERIVATIVE TEST 1ST DERIVATIVE TEST

$\underbrace{f''(-3) = 0}_{?}$, $\underbrace{f''(0) = -2}_{X=0 \text{ MAX}}$, $\underbrace{f''(2) = -1}_{X=2 \text{ MAX}}$, $f'(-4) = -12$, $f'(-1) = 5$, $f'(1) = -15$

f' NEG ↘ X=-3 ↗ f' POS   X=-3 MIN

- ✱ $f(-3)$ is a relative **minimum**, $f(0)$ is a relative **maximum**, and $f(2)$ is a relative **maximum**.
- ○ $f(-3)$ is **neither**, $f(0)$ is a relative **maximum**, and $f(2)$ is a relative **maximum**.
- ○ $f(-3)$ is a relative **minimum**, $f(0)$ is a relative **maximum**, and $f(2)$ is a relative **minimum**.
- ○ $f(-3)$ is **neither**, $f(0)$ is a relative **minimum**, and $f(2)$ is a relative **minimum**.

Question 9 3 pts

Evaluate the limit: $\lim_{x \to -\infty} \left(\frac{4x^2 - 3x - 7}{4x^3 - 7x^2 + 3x} \right) \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to -\infty} \left(\frac{\frac{4}{x} - \frac{3}{x^2} - \frac{7}{x^3}}{4 - \frac{7}{x} + \frac{3}{x^2}} \right)$

$= \frac{0 - 0 - 0}{4 - 0 + 0}$

$= \boxed{0}$

Enter your exact numerical answer in the box below.

0

Question 10

3 pts

Evaluate the limit: $\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+4}}$ $= \lim_{x\to\infty} \sqrt{\frac{x}{x+4}} = \lim_{x\to\infty} \sqrt{\frac{x}{x+4} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}}$

$= \lim_{x\to\infty} \sqrt{\frac{1}{1+\frac{4}{x}}}$

Enter your exact numerical answer in the box below.

1

$= \lim_{x\to\infty} \sqrt{\frac{1}{1+0}} = \boxed{1}$

Question 11

3 pts

What is the maximum number of horizontal asymptotes that the graph of a function can have?

ONE AS $X \to +\infty$ AND ONE AS $X \to -\infty$

- ○ Finitely many, but more than 2
- ○ 1
- ○ Infinitely many
- ● 2

Question 12

3 pts

Which one of these is the horizontal asymptote of the graph of

$R(x) = \frac{2x + x^2 + 8x^3}{3x - 4x^3}$?

$\lim_{x\to\pm\infty} R(x) = \frac{8}{-4} = -2$

$\boxed{y=-2}$

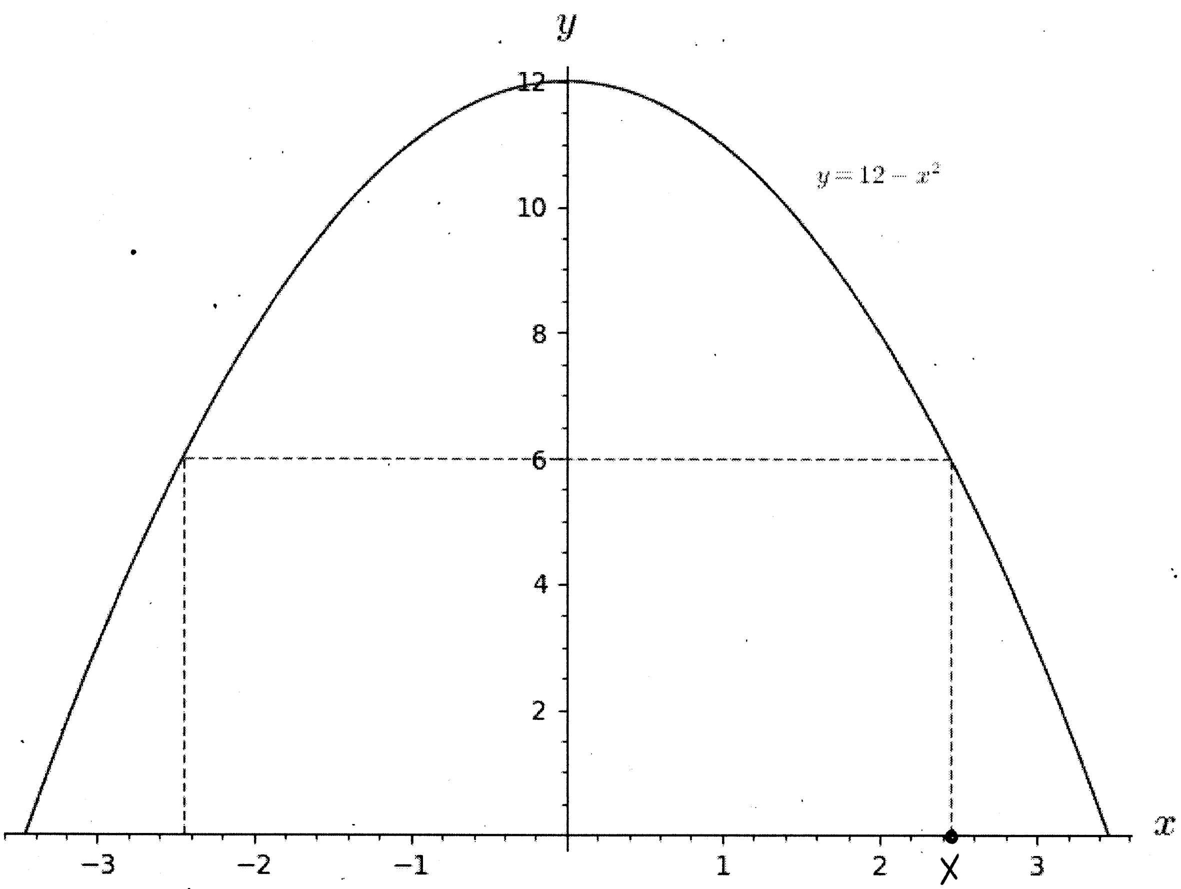
- ● $y = -2$
- ○ $x = 2/3$
- ○ $x = -2$

○ $y = 2/3$

Question 13

4 pts

The base of a rectangle lies along the x-axis with its upper two vertices on the parabola $y = 12 - x^2$. See the figure below. Using the variable x to represent the first coordinate of the point at the lower right vertex of the rectangle, find a formula for the area of the rectangle.



2x

- ✱ The area is given by $A(x) = 24x - 2x^3$.
- ○ The area is given by $A(x) = (12 - x^2)^2$.
- ○ The area is given by $A(x) = \pi x^2$.
- ○ The area is given by $A(x) = 12 - x^2$.

LENGTH $= 2x$

HEIGHT $= 12 - x^2$

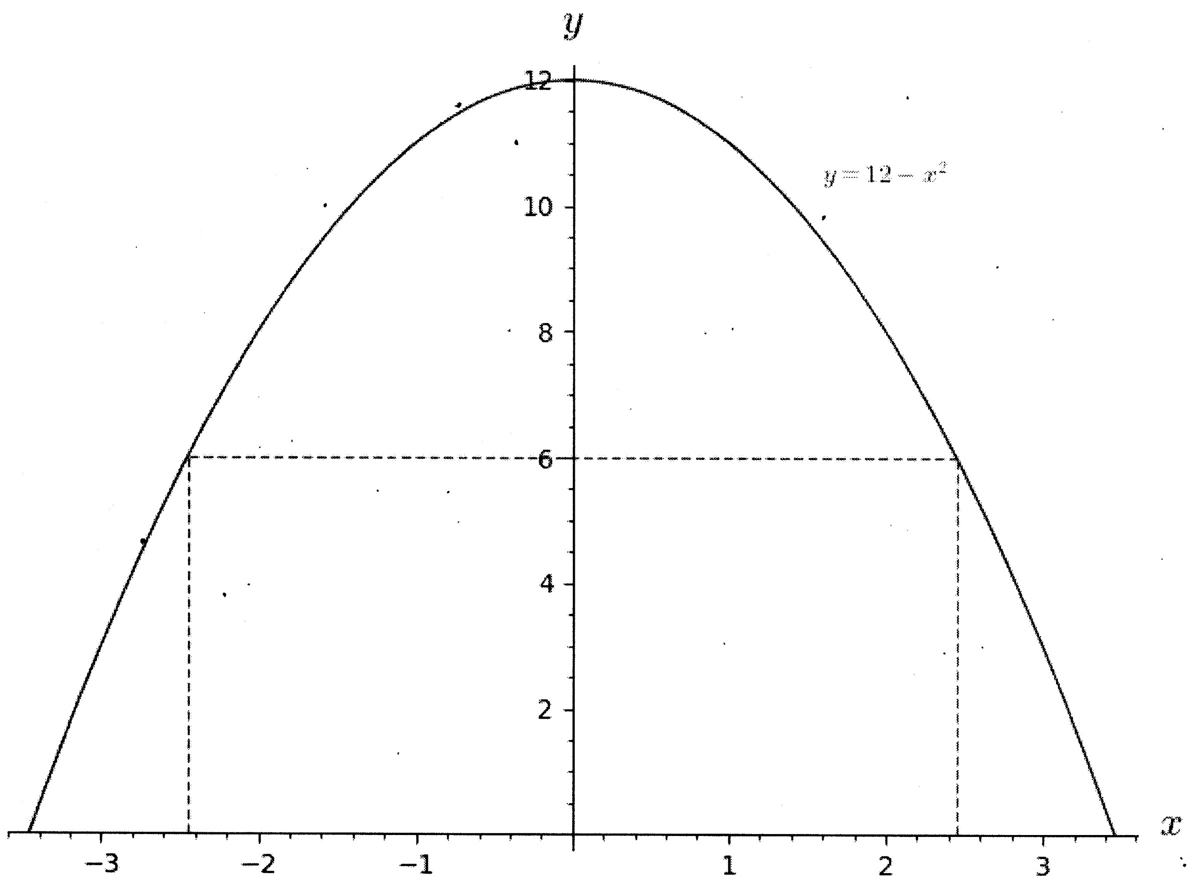
AREA $= 2x(12 - x^2)$

$= \boxed{24x - 2x^3}$

Question 14

4 pts

Refer to the problem above involving the rectangle under the parabola. The figure is duplicated below. Find the maximum area of all possible rectangles.



$A(x) = 24x - 2x^3$

$A'(x) = 24 - 6x^2 = 0 \Rightarrow x = \pm 2$

Using $x = 2$... $A(2) = 48 - 16 = \boxed{32}$

$A''(x) = -12x \quad A''(2) < 0$

$\Rightarrow$ FOUND A MAX,

- ○ 50 square units
- ○ 24 square units
- ○ 55.4 square units
- ● 32 square units

Question 15

3 pts

True or false: L'Hopital's rule can be used to evaluate a limit involving any indeterminate form.

○ True

L'HÔPITAL'S RULE APPLIES TO $\frac{0}{0}$ OR $\frac{\infty}{\infty}$.

✱ False

Question 16

6 pts

Do this problem on paper, showing all work. Then submit your solution as a pdf, jpg, or png file.

Evaluate the limit: $\lim\limits_{x \to 0} \dfrac{\tan^{-1} x}{\sin x}$

SEE ATTACHED SHEET.

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Question 17

3 pts

Which one of these forms is NOT an indeterminate form?

✱ $1/\infty$ FOR EXAMPLE, $\lim\limits_{X \to \infty} \dfrac{1}{X} = 0$

○ ∞/∞

○ 1^∞

○ $0 \cdot \infty$

Question 18

8 pts

Do this problem on paper, showing all work. Then submit your solution as a pdf, jpg, or png file.

Evaluate the limit: $\lim_{x \to 0^+} \sqrt{x} \ln x$

SEE ATTACHED SHEET.

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Question 19

4 pts

Suppose you use Newton's method to approximate a solution of $2x^3 - 3x^2 - 12x + 10 = 0$. Briefly explain why $x_0 = 2$ is NOT a good starting guess.

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THE TANGENT LINE AT $X = 2$ IS HORIZONTAL. YOU CANNOT USE IT TO GET AN X_1 VALUE. ANOTHER WAY TO SAY THIS IS THAT NEWTON'S METHOD WOULD ENCOUNTER DIVISION BY ZERO IN ITS FIRST STEP.

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Question 20

4 pts

Use Newton's method starting with $x_0 = 1$ to approximate the solution of the equation $\sin x = x - 1$. Which one of these is closest to your value of x_2?

- 2.05
- 1.87
- 2.85
- 1.93

$f(x) = x - 1 - \sin x$

$f'(x) = 1 - \cos x$

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

STARTING WITH $x_0 = 1$

$x_1 = 2.8304877...$

$x_2 = 2.049555245...$

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Question 2

$$y = e^{5x} \cos x$$

$$\frac{dy}{dx} = 5e^{5x} \cos x - e^{5x} \sin x$$

$$\boxed{dy = e^{5x}\left(5\cos x - \sin x\right) dx}$$

QUESTION 5

$g(x) = 3x - 10x^{3/5}$. g IS DEFINED FOR ALL REAL NUMBERS.

$g'(x) = 3 - 6x^{-2/5}$

$g'(x)$ DNE WHEN $x = 0$.

$g'(x) = 0 \Rightarrow x^{-2/5} = \frac{1}{2} \Rightarrow x^{2/5} = 2$

$\Rightarrow x = 2^{5/2} = \pm\sqrt{32} = \pm 4\sqrt{2}$

$\boxed{\text{CRITICAL NUMBERS ARE } x = 0,\ x = 4\sqrt{2},\ x = -4\sqrt{2}}$

Question 7

$$f(x) = x^5 - 10x^4 + 25x^3$$

$$f'(x) = 5x^4 - 40x^3 + 75x^2$$
$$= 5x^2(x^2 - 8x + 15)$$
$$= 5x^2(x-3)(x-5) = 0$$

$$\underbrace{x = 0,\ x = 3,\ x = 5}_{\text{Crit \#'s}}$$

Sign of f': $+$ $\quad$ $+$ $\quad$ $-$ $\quad$ $+$

$x = 0$, $x = 3$, $x = 5$

f is: INC ↗ $\quad$ INC ↗ $\quad$ DEC ↘ $\quad$ INC ↗

f is increasing on $(-\infty, 0) \cup (0, 3) \cup (5, \infty)$.

f is decreasing on $(3, 5)$.

$f(3) = 108$ is a rel. max.

$f(5) = 0$ is a rel. min.

$$f''(x) = 20x^3 - 120x^2 + 150x$$
$$= 10x(2x^2 - 12x + 15) = 0$$

$$\underbrace{x = 0,\ x = \frac{6-\sqrt{6}}{2},\ x = \frac{6+\sqrt{6}}{2}}_{\text{PIP's}}$$

Sign of f'': $-$ $\quad$ $+$ $\quad$ $-$ $\quad$ $+$

$x = 0$, $x = \frac{6-\sqrt{6}}{2}$, $x = \frac{6+\sqrt{6}}{2}$

Graph is: CD $\cap$ $\quad$ CU $\cup$ $\quad$ CD $\cap$ $\quad$ CU $\cup$

Graph is concave up on $\left(0, \frac{6-\sqrt{6}}{2}\right) \cup \left(\frac{6+\sqrt{6}}{2}, \infty\right)$.

Graph is concave down on $(-\infty, 0) \cup \left(\frac{6-\sqrt{6}}{2}, \frac{6+\sqrt{6}}{2}\right)$

Inflection pts at $(0,0)$, $\left(\frac{6-\sqrt{6}}{2}, 58.18\right)$, and $\left(\frac{6+\sqrt{6}}{2}, 45.32\right)$

QUESTION 16

$$\lim_{x \to 0} \frac{\tan^{-1} x}{\sin x} \qquad 0/0$$

L'HÔPITAL'S RULE

$$= \lim_{x \to 0} \frac{\frac{1}{1+x^2}}{\cos x.} = \frac{\frac{1}{1}}{1} = \boxed{1}$$

QUESTION 18

$$0 \cdot \infty$$

$$\lim_{x \to 0^+} \sqrt{x} \ln x$$

$$\infty/\infty$$

$$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} = \lim_{x \to 0^+} \frac{\ln x}{x^{-1/2}}$$

L'HÔPITAL'S RULE

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{2} x^{-3/2}}$$

$$= \lim_{x \to 0^+} \frac{-2x^{3/2}}{x} = \lim_{x \to 0^+} -2\sqrt{x} = \boxed{0}$$