# Math 131 - Quiz 10

November 15, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Let $f(x) = 4x^{1/3} - x^{4/3}$. Use the first derivative test to determine open intervals on which $f$ is increasing/decreasing and to classify the critical numbers of $f$. Then use the second derivative test to find open intervals on which the graph of $f$ is concave up/down and to determine any inflection points.

$$f'(x) = \frac{4}{3}x^{-2/3} - \frac{4}{3}x^{1/3}$$

$$= \frac{4}{3}x^{-2/3}(1-x)$$

$$\left.\begin{array}{l} f'(x) = 0 \Rightarrow x = 1 \\ f'(x) \text{ DNE} \Rightarrow x = 0 \end{array}\right\} \text{BOTH ARE CRIT \#'S.}$$

| SIGNS OF $f'$ | $+$ | | $+$ | | $-$ |
|---|---|---|---|---|---|
| | | 0 | | 1 | |
| $f$ IS | INC | | INC | | DEC |

$f$ IS INCREASING ON $(-\infty, 0) \cup (0, 1)$.

$f$ IS DECREASING ON $(1, \infty)$.

$f(0) = 0$ IS NEITHER A REL MAX NOR MIN.

$f(1) = 3$ IS A REL MAX.

$$f''(x) = -\frac{8}{9}x^{-5/3} - \frac{4}{9}x^{-2/3}$$

$$= -\frac{4}{9}x^{-5/3}(2+x)$$

$$f''(x) = 0 \Rightarrow x = -2$$

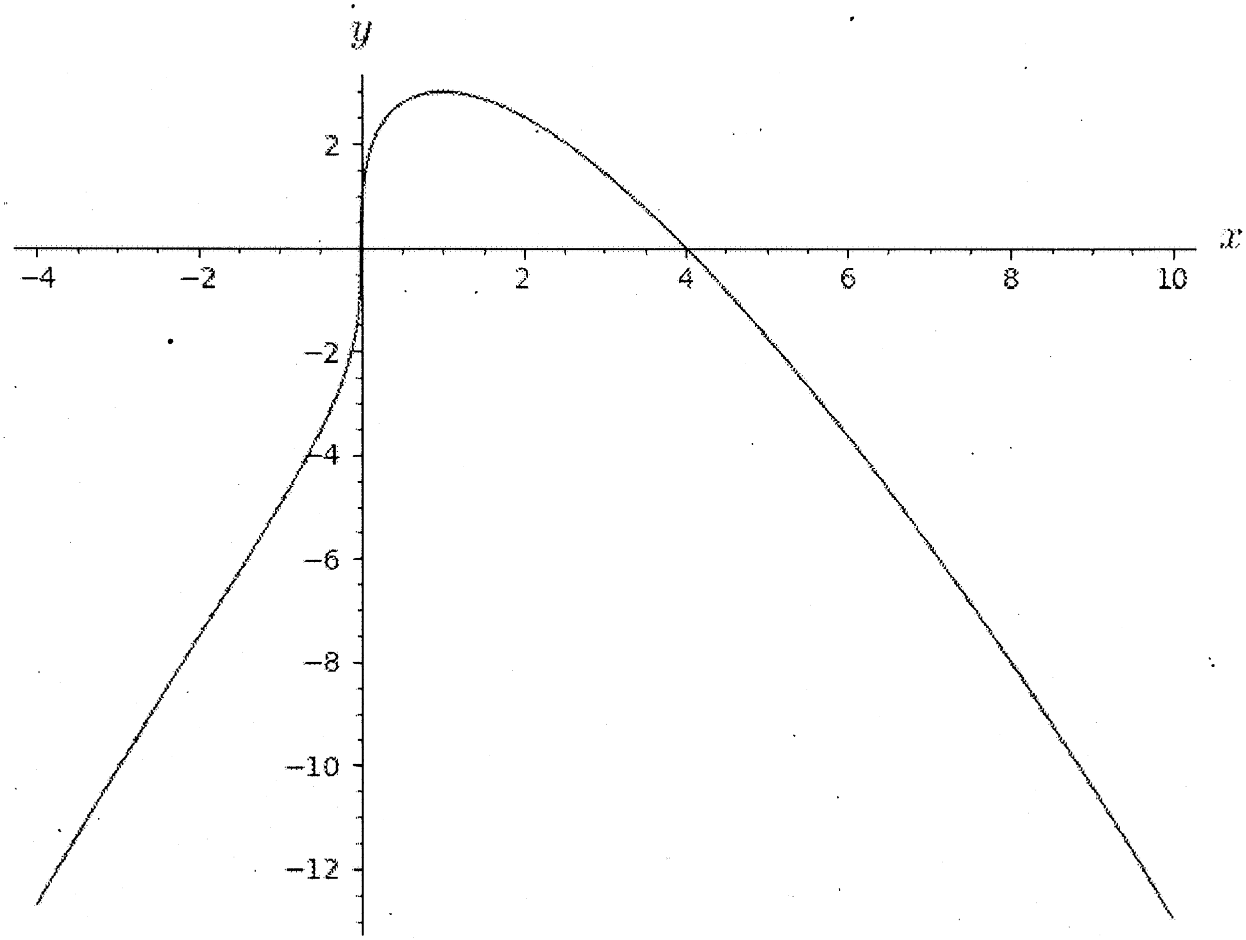
$$f''(x) \text{ DNE} \Rightarrow x = 0$$

| SIGNS OF $f''$ | $-$ | | $+$ | | $-$ |
|---|---|---|---|---|---|
| | | $-2$ | | 0 | |
| GRAPH IS | CD | | CU | | CD |

GRAPH IS CONCAVE DOWN ON $(-\infty, -2) \cup (0, \infty)$ AND CONCAVE UP ON $(-2, 0)$.

$(-2, -6\sqrt[3]{2})$ AND $(0,0)$ ARE INFLECTION PTS.

FYI: GRAPH IS ATTACHED.



y
x
2
−2
−4
−6
−8
−10
−12
−4
2
4
6
8
10