Math 131 - Quiz 12

December 6, 2023

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find the function f that satisfies $f'(x) = 9x^2 - 3x + 4\sin x$ and $f(0) = 7$.

$f(x) = \int (9x^2 - 3x + 4\sin x)\,dx = 3x^3 - \frac{3}{2}x^2 - 4\cos x + C$

$f(0) = 7 = 3(0)^3 - \frac{3}{2}(0)^2 - 4\cos(0) + C = -4 + C$

$7 = -4 + C \Rightarrow C = 11$

$f(x) = 3x^3 - \frac{3}{2}x^2 - 4\cos x + 11$

2. (3 points) Use 4 subintervals of equal length and subinterval right endpoints to compute a Riemann sum for $f(x) = \sin(x^2)$ on the interval $[0, 1]$.

$\Delta x = \frac{1-0}{4} = 0.25$

PARTITION:

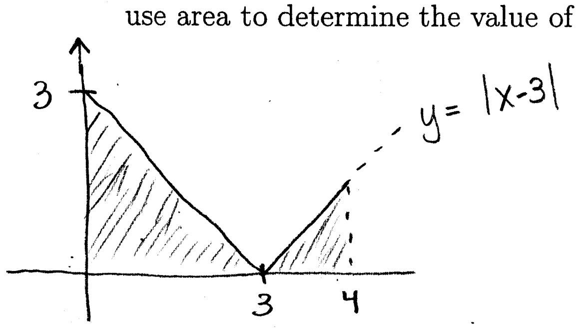
$0 < 0.25 < 0.5 < 0.75 < 1$

RIGHT ENDPTS

Riemann sum $= \sin(0.25^2)(0.25) + \sin(0.5^2)(0.25) + \sin(0.75^2)(0.25) + \sin(1^2)(0.25)$

≈ 0.421159

3. (3 points) Sketch the graph of $y = |x - 3|$ over the interval from $x = 0$ to $x = 4$. Then use area to determine the value of the definite integral $\int_0^4 |x - 3|\,dx$.



$\int_0^4 |x-3|\,dx$ = SHADED AREA (TRIANGLES)

$= \frac{1}{2}(3)(3) + \frac{1}{2}(1)(1)$

$= \frac{9}{2} + \frac{1}{2} = \boxed{5}$

4. (2 points) Use the fundamental theorem of calculus to evaluate $\int_1^4 \left(\frac{1}{\sqrt{x}} + \sqrt{x} + x + 1\right) dx$.

$\int_1^4 (x^{-1/2} + x^{1/2} + x + 1)\,dx = 2x^{1/2} + \frac{2}{3}x^{3/2} + \frac{1}{2}x^2 + x \Big|_1^4$

$= \left(2(4)^{1/2} + \frac{2}{3}(4)^{3/2} + \frac{1}{2}(4)^2 + 4\right) - \left(2 + \frac{2}{3} + \frac{1}{2} + 1\right)$

$= \frac{103}{6} = 17.1\overline{6}$