Math 131 - Quiz 4

September 20, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 25.

1. (2 points) Find the numbers a and b so that f is continuous everywhere.

$$f(x) = \begin{cases} ax^2 + bx + 6, & x < 2 \\ 10, & x = 2 \\ b + 14\cos(\pi x), & x > 2 \end{cases}.$$

THE ONLY POSSIBLE DISCONT IS AT $X = 2$...

$f(2) = 10$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (ax^2 + bx + 6)$

$= 4a + 2b + 6$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (b + 14\cos(\pi x))$

$= b + 14$

$b + 14 = 10 \Rightarrow \boxed{b = -4}$

$4a + 2b + 6 = 10$

$\Rightarrow 4a - 8 + 6 = 10$

$\Rightarrow 4a = 12$

$\Rightarrow \boxed{a = 3}$

2. (2 points) Find and classify the discontinuities of $Q(x) = \dfrac{x^2 - x - 2}{x^3 + x^2 - 6x}$.

$$Q(x) = \frac{\cancel{(x-2)}(x+1)}{\underbrace{x(x+3)\cancel{(x-2)}}}$$

DISCONTS AT $X = 0$, $X = -3$, $X = 2$.

$\boxed{X = 0: \text{ INFINITE DISCONT}}$, ASSOCIATED WITH $\frac{\text{NONZERO}}{\text{ZERO}}$ FORM

$\boxed{X = -3: \text{ INFINITE DISCONT}}$, ASSOCIATED WITH $\frac{\text{NONZERO}}{\text{ZERO}}$ FORM

Turn over.

$\boxed{X = 2: \text{ REMOVABLE DISCONT}}$, LIMIT EXISTS AT $X = 2$, LIMIT IS $\frac{3}{10}$

3. (2 points) Find an interval of length one that contains a solution of the equation $x^3 - 5x^2 + x - 10 = 0$. Use the Intermediate Value Theorem to explain your answer.

LET $p(x) = x^3 - 5x^2 + x - 10$. p IS CONTINUOUS EVERYWHERE.

$p(5) = -5$ AND $p(6) = 32$. ACCORDING TO THE IVT, $p(x)$ TAKES ON ALL y-VALUES BETWEEN -5 & 32 FOR X-VALUES BETWEEN 5 & 6.

THEREFORE, $p(x) = 0$ SOMEWHERE IN $[5, 6]$.

4. (4 points) Let $f(x) = 5 + x - x^2$.

(a) Use the limit definition of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left[5 + (x+h) - (x+h)^2\right] - \left[5 + x - x^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{5 + x + h - x^2 - 2xh - h^2 - 5 - x + x^2}{h} = \lim_{h \to 0} \frac{h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} (1 - 2x - h) = 1 - 2x$$

$$\boxed{f'(x) = 1 - 2x}$$

(b) Using your answer from part (a), find an equation of the line tangent to the graph of f at the point where $x = 2$.

$m = f'(2) = 1 - 4 = -3$

$x = 2 \Rightarrow y = f(2) = 5 + 2 - 4 = 3$

LINE: $y - 3 = -3(x - 2)$

OR

$y = -3x + 9$