Math 131 - Quiz 7

Name key

October 19, 2023

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 24.

1. (4 points) Let $f(x) = x^5 + 7x - 9$.

   (a) Compute $f^{-1}(-1)$.

$f^{-1}(-1) = y \Rightarrow f(y) = -1 \Rightarrow y^5 + 7y - 9 = -1$

$y = 1$ (Guess & check)

$\boxed{f^{-1}(-1) = 1}$

   (b) Compute $(f^{-1})'(-1)$.

$f'(x) = 5x^4 + 7$

$= \dfrac{1}{f'(f^{-1}(-1))} = \dfrac{1}{f'(1)} = \dfrac{1}{5+7} = \boxed{\dfrac{1}{12}}$

   (c) Compute $f^{-1}(11)$. (You'll probably have to use a calculator to approximate the value.)

$f^{-1}(11) = y \Rightarrow f(y) = 11 \Rightarrow y^5 + 7y - 9 = 11 \Rightarrow y^5 + 7y - 20 = 0$

$y \approx 1.55564$

$\boxed{f^{-1}(11) \approx 1.55564}$

   (d) Compute $(f^{-1})'(11)$.

$= \dfrac{1}{f'(f^{-1}(11))} \approx \dfrac{1}{f'(1.55564)} \approx \boxed{0.0276}$

Turn over.

2. (3 points) Compute the slope of the line tangent to the graph of $y = x\sin^{-1}(2x)$ at the point where $x = 1/4$. Write your answer in exact form, simplified as much as possible.

$$\frac{dy}{dx} = \sin^{-1}(2x) + x \cdot \frac{2}{\sqrt{1-(2x)^2}} \Rightarrow \frac{dy}{dx} = \sin^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$$

$$\left.\frac{dy}{dx}\right|_{x=1/4} = \sin^{-1}\left(\frac{1}{2}\right) + \frac{1/2}{\sqrt{1-\frac{4}{16}}}$$

$$= \frac{\pi}{6} + \frac{1/2}{\sqrt{3/4}} = \boxed{\frac{\pi}{6} + \frac{1}{\sqrt{3}} \approx 1.101}$$

3. (3 points) Determine each derivative.

(a) $\frac{d}{dx}\tan^{-1}(x^2+\pi)$

$$= \frac{1}{(x^2+\pi)^2+1} \cdot 2x = \boxed{\frac{2x}{1+(x^2+\pi)^2}}$$

(b) $\frac{d}{dx}(\cos^{-1}x)^2$

$$= 2(\cos^{-1}x)\frac{d}{dx}\cos^{-1}x$$

$$= 2\cos^{-1}x \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$= \boxed{\frac{-2\cos^{-1}x}{\sqrt{1-x^2}}}$$