Math 131 - Test 1

September 13, 2023

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. Determine all limits analytically unless otherwise indicated. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (4 points) Suppose the function f is defined on an open interval around the number 2. Describe what the following statement means.

$$\lim_{x\to 2} f(x) = 9$$

IT MEANS THAT THE VALUES OF $f(x)$ GET CLOSER AND CLOSER TO 9 AS THE VALUES OF X GET CLOSER AND CLOSER TO 2. IN FACT, THE VALUES OF $f(x)$ CAN BE MADE ARBITRARILY CLOSE TO 9 BY CHOOSING X CLOSE ENOUGH TO 2. (SEE LECTURE 3 NOTES.)

2. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x\to 2} \frac{1-3^{x-2}}{2x-4}$$

$$f(x) = \frac{1-3^{x-2}}{2x-4}$$

X	f(x)
1.9	-0.520208
1.99	-0.546300
1.999	-0.549005
2.1	-0.580616
2.01	-0.552335
2.001	-0.549608

IT LOOKS LIKE THE LIMIT IS ABOUT -0.55.

THE EXACT LIMIT IS $-\frac{\ln 3}{2} = -0.549306144\ldots$

3. (4 points) Give an example of a limit that fails to exist and say why it fails to exist.

HERE ARE SEVERAL ...

① $\lim_{x\to 0} \frac{|x|}{x}$ DNE

BECAUSE

$\lim_{x\to 0^+} \frac{|x|}{x} = 1 \neq \lim_{x\to 0^-} \frac{|x|}{x} = -1$

② $\lim_{x\to 0} \sqrt{x}$ DNE

BECAUSE $\sqrt{x}$ IS NOT DEFINED FOR $x<0$

③ $\lim_{x\to 0} \frac{1}{x^2} = +\infty$

DNE, UNBOUNDED GROWTH.

4. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{t\to 0} \frac{(t+6)^2-36}{t}$ 0/0 MORE WORK

$$\lim_{t\to 0} \frac{t^2+12t+36-36}{t} = \lim_{t\to 0} \frac{t^2+12t}{t} = \lim_{t\to 0} (t+12)$$
$$= \boxed{12}$$

0/0 MORE WORK

(b) $\lim_{x\to 0} \frac{\tan 3x}{5x}$

$$= \lim_{x\to 0} \frac{\sin 3x}{5x \cos 3x} = \lim_{x\to 0} \left(\frac{3}{5}\ \frac{\sin 3x}{3x}\ \frac{1}{\cos 3x}\right)$$
$$= \left(\frac{3}{5}\right)(1)(1) = \boxed{\frac{3}{5}}$$

0/0 MORE WORK

(c) $\lim_{r\to 4^-} \left(\frac{r^2-r-12}{r^2-16}\right)$

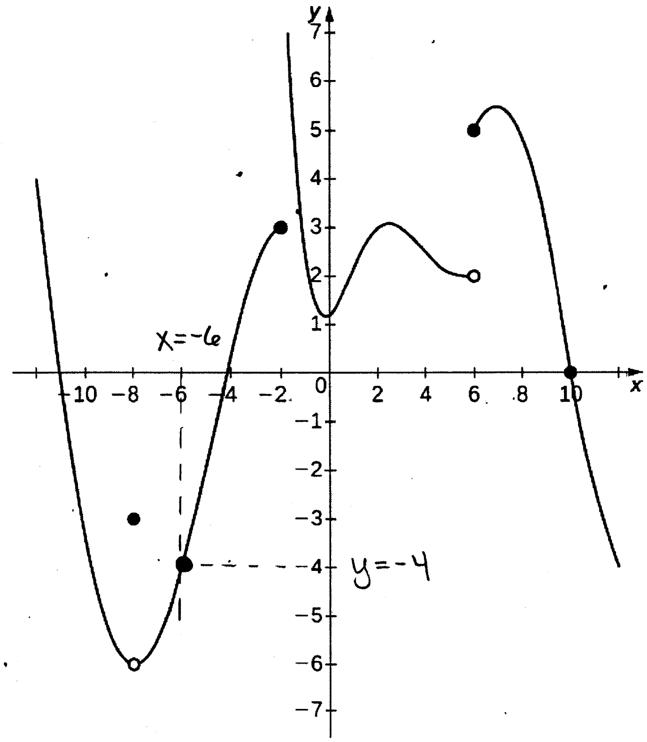
$$= \lim_{r\to 4^-} \frac{\cancel{(r-4)}(r+3)}{\cancel{(r-4)}(r+4)} = \lim_{r\to 4^-} \frac{r+3}{r+4} = \boxed{\frac{7}{8}}$$

0/0 MORE WORK

(d) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

$$= \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{x-9}}$$
$$= \lim_{x\to 9} (\sqrt{x}+3) = 3+3 = \boxed{6}$$

5. (12 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -8} f(x)$ = $\boxed{-6}$

(b) $f(6)$ = $\boxed{5}$

(c) $\lim_{x \to -2+} f(x)$ = $\boxed{+\infty}$, ASSUMING THERE IS A V.A. AT X = -2

(d) $\lim_{x \to 0} f(x)$ ≈ $\boxed{1.25}$

(e) $\lim_{x \to 6-} f(x)$ = $\boxed{2}$

(f) $\lim_{x \to -6} f(x)$ = $\boxed{-4}$

6. (3 points) Refer to the function $y = f(x)$ whose graph is shown above. Choose any number a for which it is true that $f(a) = \lim_{x \to a} f(x)$. Write your number a and the limit at $x = a$.

a = -6 IS SUCH A NUMBER

$\lim_{X \to -6} f(x) = -4$, WHICH IS EQUAL TO $f(-6)$.

THERE ARE INFINITELY MANY CHOICES FOR a.

$$x(7-x) = -x(x-7)$$

7. (8 points) Let $f(x) = \dfrac{\overbrace{7x - x^2}}{|x-7|}$.

(a) Compute the limit: $\lim_{x\to 7+} f(x)$

$x > 7$ $\quad \lim_{x\to 7^+} \dfrac{x(7-x)}{x-7} = \lim_{x\to 7+} (-x) = \boxed{-7}$

(b) Compute the limit: $\lim_{x\to 7^-} f(x)$

$x < 7$ $\quad \lim_{x\to 7^-} \dfrac{x(7-x)}{-(x-7)} = \lim_{x\to 7^-} x = \boxed{7}$

(c) What do the results of parts (a) and (b) tell you about $\lim_{x\to 7} f(x)$?

$\lim_{x\to 7} f(x)$ DNE, LEFT LIMIT $\neq$ RIGHT LIMIT

8. (16 points) In each problem below, determine analytically whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

ALL OF THESE PRODUCE A $\frac{2}{0}$ FORM. SO, EVERY ONE SHOWS SOME KIND OF UNBOUNDED GROWTH.

(a) $\lim_{x\to 2^-} \dfrac{x}{x-2} = -\infty$

TO LEFT OF $X=2$...

$\dfrac{X}{X-2} = \dfrac{+}{-} = -$ $\Rightarrow$ LIMIT MUST BE $-\infty$.

(b) $\lim_{x\to 2+} \dfrac{x}{x-2} = +\infty$

TO THE RIGHT OF $X=2$...

$\dfrac{X}{X-2} = \dfrac{+}{+} = +$ $\Rightarrow$ LIMIT MUST BE $+\infty$

(c) $\lim_{x\to 2} \dfrac{x}{x-2}$

$\boxed{\text{LIMIT DNE}}$ SEE (a) & (b).

(d) $\lim_{x\to 2} \dfrac{x}{(x-2)^4} = +\infty$

ON BOTH SIDES OF $X=2$...

$\dfrac{X}{(X-2)^4} = \dfrac{+}{+} = +$ $\Rightarrow$ LIMIT MUST BE $+\infty$.

9. (5 points) Determine the value of the constant k so that $\lim_{x \to 4} g(x)$ exists.

$$g(x) = \begin{cases} kx + \sin(\pi x), & x \le 4 \\ x\cos(\pi x) - x^2, & x > 4 \end{cases}$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4} (kx + \sin \pi x) = 4k + \sin 4\pi = 4k$$

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4} (x \cos \pi x - x^2) = 4 \cos 4\pi - 16 = -12$$

$$4k = -12 \Rightarrow \boxed{k = -3}$$

10. (5 points) Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2 + 2x - 8}{x^2 - 4}$.
(You can use your graphing calculator for help, but you must show computational work for full credit.)

$$h(x) = \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{x+4}{x+2}, \; x \neq 2$$

GRAPH HAS A HOLE AT $X=2$, BUT NOT A V.A.

AT $x=-2$, WE GET THE FORM $\frac{2}{0}$ $\Rightarrow$ $\boxed{X=-2 \text{ IS A V.A.}}$

IT IS THE ONLY V.A.

11. (3 points) Suppose that the function f satisfies

$$1 - x \le f(x) \le 1 - x + \frac{x^2}{2}$$

for all x-values. Determine the limit, $\lim_{x \to 0} f(x)$, and explain your reasoning.

By Squeeze Theorem,

$$\lim_{x \to 0} (1-x) \le \lim_{x \to 0} f(x) \le \lim_{x \to 0} \left(1 - x + \frac{x^2}{2}\right)$$

$$1 \le \lim_{x \to 0} f(x) \le 1$$

$\boxed{\text{LIMIT IS } 1}$

12. (2 points) Suppose $\lim_{x \to 2} f(x) = 17$. Which one of these statements must be true?

    (a) $f(2) = 17$

    (b) The function f must be defined at $x = 2$.

    (c) The domain of f cannot include the number 2.

    (d) The domain of f must include some numbers greater than 2.

13. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x \to 3} f(x) = -\infty$?

    (a) Direct substitution results in division by zero.

    (b) The limit at $x = 3$ does not exist because the values of f grow negatively without bound as $x \to 3$.

    (c) The limit at $x = 3$ exists, and it is a very large negative number.

    (d) The limit at $x = 3$ does not exist because $f(3)$ is not defined.

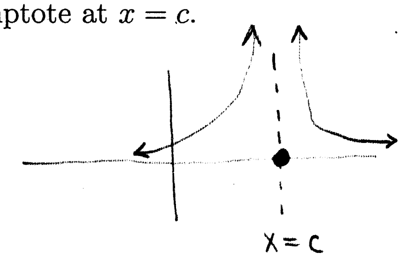
14. (2 points) Suppose $\lim_{x \to c} f(x) = \infty$. Which one of the following is NOT necessarily true?

    (a) The graph of f has a vertical asymptote at $x = c$.

    (b) $\lim_{x \to c^+} f(x) = \infty$

    (c) $\lim_{x \to c^-} f(x) = \infty$

    (d) f is not defined at $x = c$.



$f(c) = 0$

15. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x \to 5} f(x)$. Which list of x-values shown below would be best for your table?

    (a) $x = 5.01, 5.001, 5.0001,$ ~~5~~, $4.99, 4.999, 4.9999$

    (b) $x = 4.0, 4.5, 4.75,$ ~~5.0~~, $5.25, 5.5, 6.0$

    (c) $x = 5.01, 5.001, 5.0001, 4.99, 4.999, 4.9999$

    (d) $x = 4.9, 4.99, 4.999, 4.9999, 5.1$ ← NOT ENOUGH ON THE RIGHT

16. (2 points) Which of these is NOT a reason that a limit may not exist?

    (a) The one-sided limits exist, but are different. #1

    (b) The function is not defined at the limit point.

    (c) The function values grow without bound as the limit point is approached. #2

    (d) The function is not defined to the left of the limit point.

#4