Math 131 - Test 1

Name _____

September 14, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary. Determine all limits analytically unless otherwise indicated. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

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1. (4 points) Suppose the function f is defined on an open interval around the number 2. Describe what the following statement means.

$$\lim_{x \to 2} f(x) = 9$$

2. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x \to 2} \frac{1 - 3^{x-2}}{2x - 4}$$

3. (4 points) Give an example of a limit that fails to exist and say why it fails to exist.

4. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

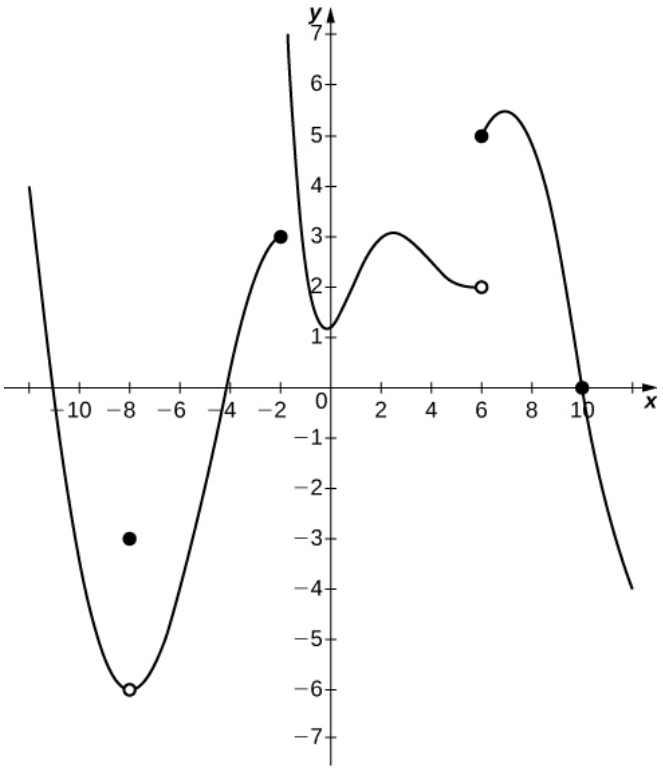
(a) $\lim_{t\to 0} \dfrac{(t+6)^2 - 36}{t}$

(b) $\lim_{x\to 0} \dfrac{\tan 3x}{5x}$

(c) $\lim_{r\to 4^-} \left(\dfrac{r^2 - r - 12}{r^2 - 16}\right)$

(d) $\lim_{x\to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

5. (12 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -8} f(x)$

(b) $f(6)$

(c) $\lim_{x \to -2^+} f(x)$

(d) $\lim_{x \to 0} f(x)$

(e) $\lim_{x \to 6^-} f(x)$

(f) $\lim_{x \to -6} f(x)$

6. (3 points) Refer to the function $y = f(x)$ whose graph is shown above. Choose any number a for which it is true that $f(a) = \lim_{x \to a} f(x)$. Write your number a and the limit at $x = a$.

7. (8 points) Let $f(x) = \dfrac{7x - x^2}{|x - 7|}$.

   (a) Compute the limit: $\lim\limits_{x \to 7^+} f(x)$

   (b) Compute the limit: $\lim\limits_{x \to 7^-} f(x)$

   (c) What do the results of parts (a) and (b) tell you about $\lim\limits_{x \to 7} f(x)$?

8. (16 points) In each problem below, determine analytically whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

   (a) $\lim\limits_{x \to 2^-} \dfrac{x}{x - 2}$

   (b) $\lim\limits_{x \to 2^+} \dfrac{x}{x - 2}$

   (c) $\lim\limits_{x \to 2} \dfrac{x}{x - 2}$

   (d) $\lim\limits_{x \to 2} \dfrac{x}{(x - 2)^4}$

9. (5 points) Determine the value of the constant k so that $\lim_{x\to 4} g(x)$ exists.

$$g(x) = \begin{cases} kx + \sin(\pi x), & x \leq 4 \\ x\cos(\pi x) - x^2, & x > 4 \end{cases}$$

10. (5 points) Determine all vertical asymptotes of the graph of $h(x) = \dfrac{x^2 + 2x - 8}{x^2 - 4}$. (You can use your graphing calculator for help, but you must show computational work for full credit.)

11. (3 points) Suppose that the function f satisfies

$$1 - x \leq f(x) \leq 1 - x + \frac{x^2}{2}$$

for all x-values. Determine the limit, $\lim_{x\to 0} f(x)$, and explain your reasoning.

12. (2 points) Suppose $\lim_{x \to 2} f(x) = 17$. Which one of these statements must be true?

    (a) $f(2) = 17$

    (b) The function f must be defined at $x = 2$.

    (c) The domain of f cannot include the number 2.

    (d) The domain of f must include some numbers greater than 2.

13. (2 points) Which one of the following best describes the meaning of the statement $\lim_{x \to 3} f(x) = -\infty$?

    (a) Direct substitution results in division by zero.

    (b) The limit at $x = 3$ does not exist because the values of f grow negatively without bound as $x \to 3$.

    (c) The limit at $x = 3$ exists, and it is a very large negative number.

    (d) The limit at $x = 3$ does not exist because $f(3)$ is not defined.

14. (2 points) Suppose $\lim_{x \to c} f(x) = \infty$. Which one of the following is NOT necessarily true?

    (a) The graph of f has a vertical asymptote at $x = c$.

    (b) $\lim_{x \to c^+} f(x) = \infty$

    (c) $\lim_{x \to c^-} f(x) = \infty$

    (d) f is not defined at $x = c$.

15. (2 points) Suppose you were asked to use a table of values to estimate $\lim_{x \to 5} f(x)$. Which list of x-values shown below would be best for your table?

    (a) $x = 5.01, 5.001, 5.0001, 5, 4.99, 4.999, 4.9999$

    (b) $x = 4.0, 4.5, 4.75, 5.0, 5.25, 5.5, 6.0$

    (c) $x = 5.01, 5.001, 5.0001, 4.99, 4.999, 4.9999$

    (d) $x = 4.9, 4.99, 4.999, 4.9999, 5.1$

16. (2 points) Which of these is NOT a reason that a limit may not exist?

    (a) The one-sided limits exist, but are different.

    (b) The function is not defined at the limit point.

    (c) The function values grow without bound as the limit point is approached.

    (d) The function is not defined to the left of the limit point.