

Math 131 - Final Exam
December 13, 2023

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \rightarrow 2} \frac{x^3 - x^2 - 2x}{3x - 6}$

(b) $\lim_{x \rightarrow 5^-} \frac{x^2 + 25}{x - 5}$

2. (10 points) Each function given below has a single point of discontinuity, and each discontinuity has a specific name. State the x -value at which the function is discontinuous and classify the discontinuity.

(a) $f(x) = \frac{\sin x}{x}$

(b) $g(x) = \frac{(x^2 + 2x)|x - 2|}{x - 2}$

(c) $h(x) = \frac{x^2 - 16}{x + 3}$

3. (10 points) Let $f(x) = 3x^2 + 2x$. Use the **limit definition of the derivative** to determine $f'(x)$. Show all work.

4. (10 points) Use basic differentiation rules to determine each derivative. Do not simplify.

(a) $\frac{d}{dt}(e^{4t} \tan^{-1} t)$

(b) $\frac{d}{dx} \sqrt[3]{x^2 + 2x + 1}$

5. (10 points) An object is thrown upward in such a way that its height after t seconds is given by $s(t) = -16t^2 + 32t + 128$, where s is measured in feet.

(a) What is the maximum height of the object?

(b) What is the object's velocity when it hits the ground?

6. (10 points) Let $f(x) = 7 + 2x^2 - 2x^3$.

(a) Use the 1st derivative test to determine all relative extreme values.

(b) Use the 2nd derivative test to determine open intervals on which the graph of f is concave up/down.

7. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{x \rightarrow \infty} \left(\frac{\ln x^4}{x^3} \right)$

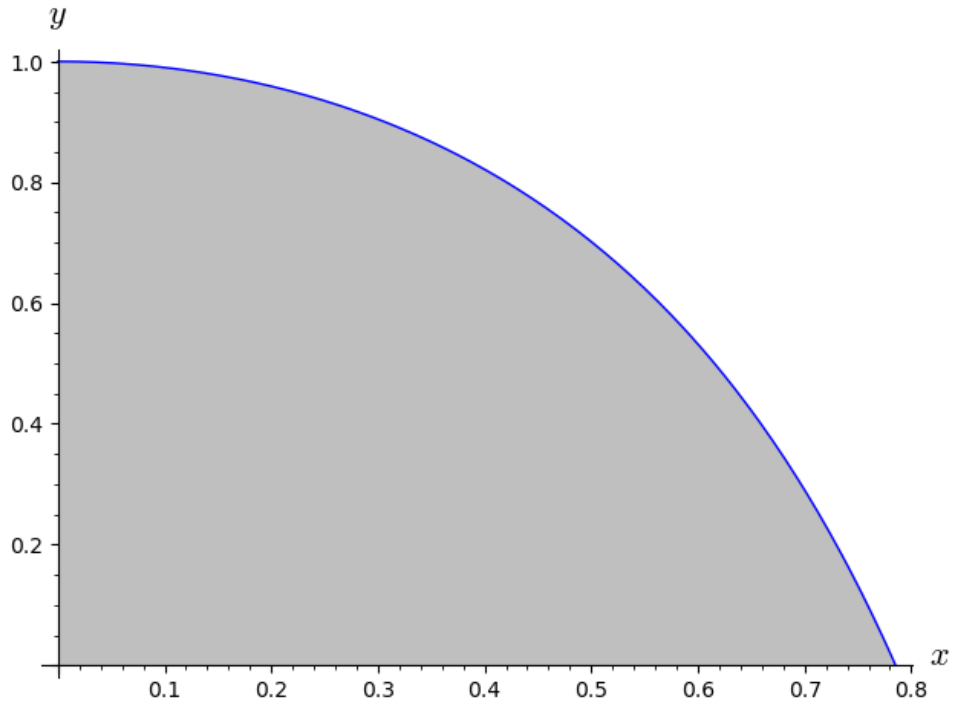
(b) $\lim_{x \rightarrow 0} \left(\frac{x^2 - 2 + 2 \cos x}{x^3} \right)$

8. (10 points) Evaluate each definite integral.

(a) $\int_1^2 \left(e^x + \frac{1}{x} \right) dx$

(b) $\int_0^{16} (\sqrt{x} + \sqrt[4]{x}) dx$

9. (10 points) The graph of $f(x) = 2 - \sec^2 x$ on $[0, \pi/4]$ is shown below, and the region under the graph is shaded.



- (a) Check that $f(0) = 1$ and $f(\pi/4) = 0$ as the graph indicates.
- (b) Use the fundamental theorem of calculus to find the area of the shaded region.

10. (10 points)

(a) Use an appropriate substitution to evaluate the indefinite integral $\int 6t^2(t^3+1)^4 dt$.

(b) Use your result from part (a) to determine the value of $\int_0^2 6t^2(t^3 + 1)^4 dt$.