Math 131 - Final Exam

December 14, 2023

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit. You may need to use $+\infty$, $-\infty$, or DNE.

   (a) $\displaystyle\lim_{x\to 2} \frac{x^3 - x^2 - 2x}{3x - 6}$

   (b) $\displaystyle\lim_{x\to 5^-} \frac{x^2 + 25}{x - 5}$

2. (10 points) Each function given below has a single point of discontinuity, and each discontinuity has a specific name. State the x-value at which the function is discontinuous and classify the discontinuity.

   (a) $f(x) = \dfrac{\sin x}{x}$

   (b) $g(x) = \dfrac{(x^2 + 2x)|x - 2|}{x - 2}$

   (c) $h(x) = \dfrac{x^2 - 16}{x + 3}$

3. (10 points) Let $f(x) = 3x^2 + 2x$. Use the **limit definition of the derivative** to determine $f'(x)$. Show all work.

4. (10 points) Use basic differentiation rules to determine each derivative. Do not simplify.

(a) $\dfrac{d}{dt}(e^{4t} \tan^{-1} t)$

(b) $\dfrac{d}{dx} \sqrt[3]{x^2 + 2x + 1}$

5. (10 points) An object is thrown upward in such a way that its height after t seconds is given by $s(t) = -16t^2 + 32t + 128$, where s is measured in feet.

   (a) What is the maximum height of the object?

   (b) What is the object's velocity when it hits the ground?

6. (10 points) Let $f(x) = 7 + 2x^2 - 2x^3$.

   (a) Use the 1st derivative test to determine all relative extreme values.

   (b) Use the 2nd derivative test to determine open intervals on which the graph of f is concave up/down.

7. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{x \to \infty} \left(\frac{\ln x^4}{x^3} \right)$

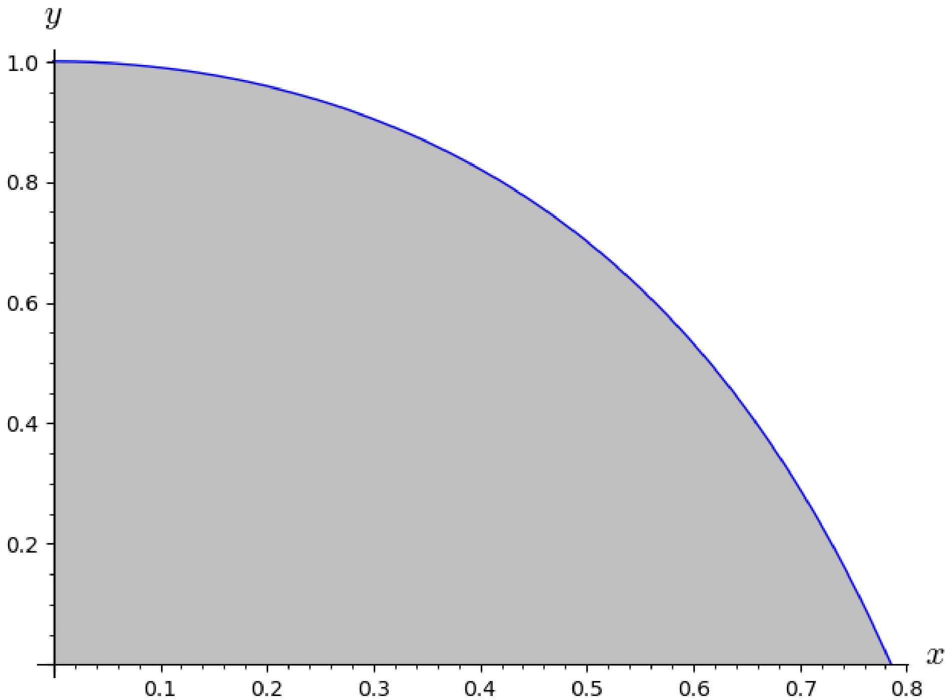
(b) $\lim_{x \to 0} \left(\frac{x^2 - 2 + 2\cos x}{x^3} \right)$

8. (10 points) Evaluate each definite integral.

(a) $\int_1^2 \left(e^x + \frac{1}{x} \right) dx$

(b) $\int_0^{16} (\sqrt{x} + \sqrt[4]{x})\, dx$

9. (10 points) The graph of $f(x) = 2 - \sec^2 x$ on $[0, \pi/4]$ is shown below, and the region under the graph is shaded.



(a) Check that $f(0) = 1$ and $f(\pi/4) = 0$ as the graph indicates.

(b) Use the fundamental theorem of calculus to find the area of the shaded region.

10. (10 points)

(a) Use an appropriate substitution to evaluate the indefinite integral $\int 6t^2(t^3+1)^4\,dt$.

(b) Use your result from part (a) to determine the value of $\int_0^2 6t^2(t^3+1)^4\,dt$.