# Math 131 - Quiz 4

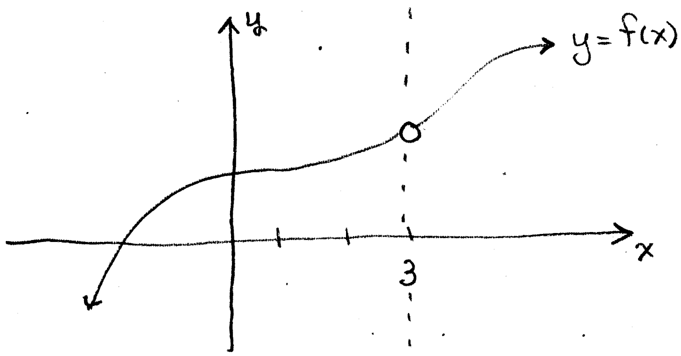
September 18, 2025

Name key

Score ________

Show all work to receive credit. Supply explanations where necessary. The take-home problem is due September 23.

1. (2 points) Sketch the graph of a function that has a limit at $x = 3$, but is not continuous at $x = 3$. Then classify the discontinuity.




$f(3)$ IS NOT DEFINED.

THE DISCONT AT $X = 3$ IS REMOVABLE BECAUSE $\lim\limits_{x \to 3} f(x)$ EXISTS.

2. (2 points) Find and classify the discontinuities of $g(x) = \dfrac{4\cos x + 9\sin x}{x^2 - 9}$. You must show work that supports your answer.

$g$ IS CONTINUOUS EVERYWHERE THAT IT IS DEFINED, AND IT'S NOT DEFINED WHEN

$x^2 - 9 = (x+3)(x-3) = 0$

$\boxed{\text{DISCONTS AT } X = 3 \text{ AND } X = -3}$

BOTH ARE ASSOCIATED WITH $\frac{k \neq 0}{0}$ WHEN SUBSTITUTED.

BOTH ARE INFINITE DISCONTS.

3. (3 points) Find the number $k$ that makes $f$ continuous everywhere. For full credit, you work must show how you are using limits and the definition of continuity.

$$f(x) = \begin{cases} x + 6\cos(\pi x), & x \le 2 \\ kx^2 - 2x + 5, & x > 2 \end{cases}$$

$\lim\limits_{x \to 2} f(x) = f(2)$

FOR ANY $k$, $f$ IS CONTINUOUS EVERYWHERE WITH THE POSSIBLE EXCEPTION OF $X = 2$.

WE NEED ONLY LOOK AT $X = 2$.

$f(2) = 2 + 6\cos(2\pi) = 8$

$\lim\limits_{x \to 2^-} f(x) = 2 + 6\cos(2\pi) = 8$

$\lim\limits_{x \to 2^+} f(x) = k(2)^2 - 2(2) + 5 = 4k + 1$

TO MAKE $f$ CONTINUOUS AT $X = 2$, WE NEED $4k + 1 = 8$

$4k = 7$

$\boxed{k = \frac{7}{4}}$

4. (3 points) Use the limit definition of derivative to find $f'(x)$. Show all work.

$$f(x) = 1 + 5x - x^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[1+5(x+h) - (x+h)^2] - [1+5x-x^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{5x} + 5h - \cancel{x^2} - 2xh - h^2 - \cancel{1} - \cancel{5x} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{5h - 2xh - h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(5-2x-h)}{\cancel{h}}$$

$$= \boxed{5-2x}$$