

Math 131 - Test 1
September 11, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (8 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$$\lim_{x \rightarrow 3} \frac{1 - x}{x^2 - 3x}$$

2. (9 points) Given the following information,

$$f(1) = 3, \quad \lim_{x \rightarrow 1} f(x) = -2, \quad g(1) = 4, \quad \lim_{x \rightarrow 1} g(x) = 12,$$

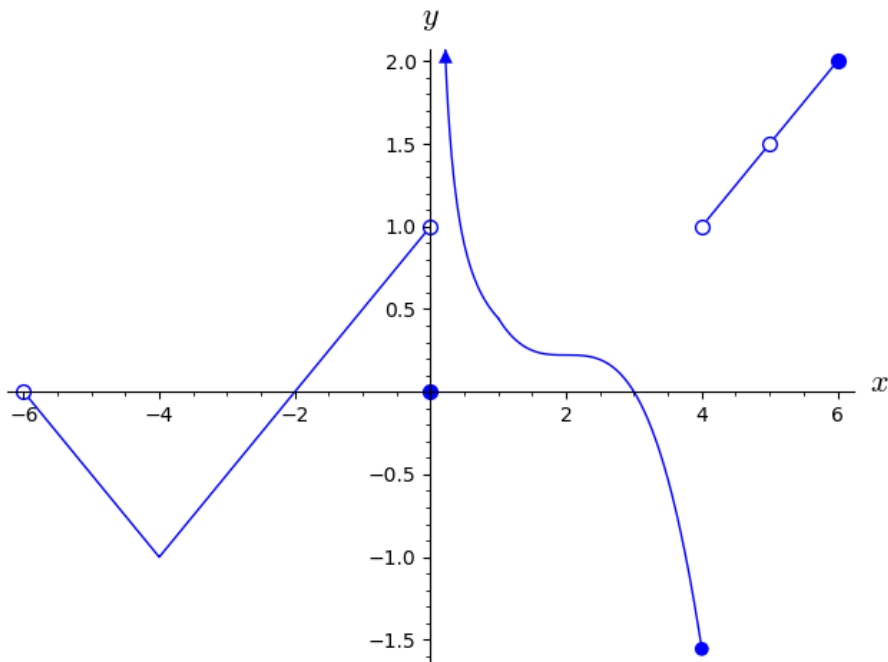
find the value of each expression below. To receive credit, you must show how you used the limit laws.

(a) $\lim_{x \rightarrow 1} (8f(x) - 5g(x))$

(b) $\lim_{x \rightarrow 1} \frac{f(x)g(x)}{x + 1}$

(c) $\sqrt{g(1)} + \lim_{x \rightarrow 1} (x + f(x))^3$

3. (12 points) Referring to the graph of $y = f(x)$ shown below, estimate each of the following or explain why it does not exist.



(a) $\lim_{x \rightarrow -4} f(x)$

(b) $\lim_{x \rightarrow 0^+} f(x)$

(c) $\lim_{x \rightarrow 0^-} f(x)$

(d) $\lim_{x \rightarrow 5} f(x)$

(e) $\lim_{x \rightarrow 4} f(x)$

(f) $\lim_{x \rightarrow 6} f(x)$

4. (7 points) The function $y = h(x)$ is defined below.

$$h(x) = \begin{cases} |4x| + \sin(\pi x), & x < 2 \\ 3x^2 - 5x + 1, & x \geq 2 \end{cases}$$

Find each limit analytically. If the limit does not exist, you must say why.

(a) $\lim_{x \rightarrow 1} h(x)$

(b) $\lim_{x \rightarrow 5} h(x)$

(c) $\lim_{x \rightarrow 2} h(x)$

5. (10 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist. Show work that supports your answer.

(a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x}}{x}$

(b) $\lim_{x \rightarrow 9} \frac{x^2 - 9x}{|x - 9|}$

(c) $\lim_{x \rightarrow 0} g(x)$, where $g(x) = \begin{cases} x + 2, & 0 \leq x < 1 \\ 5x + 7, & x \geq 1 \end{cases}$

6. (24 points) **Determine each limit analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x^2 - 1}$

(b) $\lim_{u \rightarrow 0} \frac{(u - 2)^2 - 4}{u}$

(c) $\lim_{x \rightarrow 0} \frac{(x + 3) \sin 2x}{4x}$

(d) $\lim_{t \rightarrow 9} \frac{3 - \sqrt{t}}{18 - 2t}$

7. (4 points) Suppose $f(x)$ is a function for which

$$3x \leq f(x) \leq x^3 + 2$$

whenever $0 < x < 2$. Compute $\lim_{x \rightarrow 1} f(x)$ and explain your reasoning.

8. (12 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \rightarrow 5} \frac{\sqrt{x}}{|x - 5|}$

(b) $\lim_{x \rightarrow 2^-} \left(\frac{7x}{2 - x} \right)$

(c) $\lim_{x \rightarrow \pi^+} \left(\frac{3}{\sin x} \right)$

(d) $\lim_{x \rightarrow 1} \left[\frac{x - 5}{x - 1} \right]$

9. (4 points) Determine all vertical asymptotes of the graph of $R(x) = \frac{x^2 + 2x}{x^3 - 4x}$.

10. (2 points) When evaluating the limit of a rational function, you tried direct substitution and obtained a nonzero over zero form. Which of these must be true?
- (a) You must use L'Hôpital's rule to determine the limit.
 - (b) The limit does not exist because the function values grow without bound around the limit point.
 - (c) The limit exists, but could be any number.
 - (d) No conclusion can be drawn from that form.
11. (2 points) The function f is defined for all real numbers, and $f(2) = 5$. Which one of these statements must be true?
- (a) $\lim_{x \rightarrow 2} f(x)$ exists.
 - (b) $\lim_{x \rightarrow 2} f(x)$ does not exist.
 - (c) $\lim_{x \rightarrow 2} f(x) = 5$
 - (d) Nothing can be said about $\lim_{x \rightarrow 2} f(x)$ without more information.
12. (2 points) Suppose the graph of f has a vertical asymptote at $x = -2$. Which of these cannot be true?
- (a) $f(-2) = 13$
 - (b) $\lim_{x \rightarrow -2} f(x) = 1$
 - (c) $\lim_{x \rightarrow -2^+} f(x) = 7$
 - (d) $\lim_{x \rightarrow -2} f(x) = -\infty$
13. (2 points) f is a polynomial function, and you would like to find $\lim_{x \rightarrow 0} f(x)$. Which one of these is false?
- (a) You can find the limit by direct substitution.
 - (b) It is impossible to obtain a $0/0$ form by direct substitution.
 - (c) $\lim_{x \rightarrow 0} f(x) = f(0)$
 - (d) f is not defined on an interval around $x = 0$.
14. (2 points) Which one of these is a possible description of a rational function?
- (a) A polynomial divided by another polynomial.
 - (b) A polynomial divided by a trigonometric function.
 - (c) A product of an exponential function and a polynomial.
 - (d) A quotient of a radical function and an exponential function.