Math 131 - Test 2
October 9, 2025

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, use differentiation rules for all derivatives and do not simplify.

1. (8 points) Let $f(x) = 4 - x + 2x^2$. Use the limit definition of derivative (not differentiation rules) to determine $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[4-(x+h)+2(x+h)^2]-[4-x+2x^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4}-\cancel{x}-h+\cancel{2x^2}+4xh+2h^2-\cancel{4}+\cancel{x}-\cancel{2x^2}}{}$$

$$= \lim_{h\to 0} \frac{-h+4xh+2h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-1+4x+2h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (-1+4x+2h) = -1+4x$$

$$\boxed{f'(x) = -1+4x}$$

2. (6 points) Use differentiation rules to confirm your result above. Then find an equation of the line tangent to the graph of $f(x) = 4 - x + 2x^2$ at the point where $x = -1$. Write your final answer in slope-intercept form.

$$f'(x) = 0 - 1 + 2(2x^1)$$
$$= -1+4x \checkmark$$

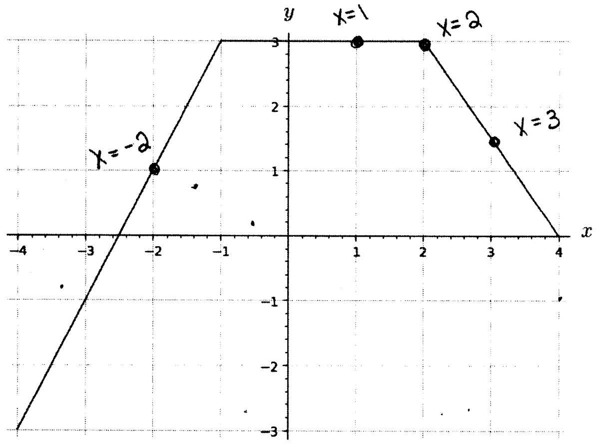
Slope $= f'(-1) = -5$

Point: $x = -1$
$y = f(-1) = 4+1+2 = 7$

$y - 7 = -5(x+1)$

$$\boxed{y = -5x + 2}$$

3. (6 points) The graph of the function f is shown below. Use the graph to determine each of the following. Show work or explain.



(a) $f'(-2)$ = SLOPE AT $X=-2$ = $\boxed{2}$ $\frac{\text{RISE}}{\text{RUN}} = \frac{2}{1}$

(b) $f'(3)$ = SLOPE AT $X=3$ = $\boxed{-\frac{3}{2}}$ $\frac{\text{RISE}}{\text{RUN}} = \frac{-3}{2}$

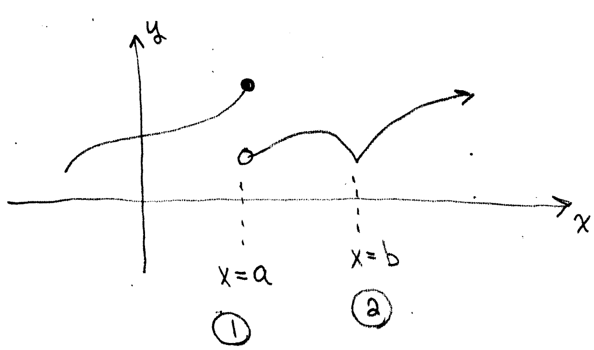
(c) $f'(1)$ = SLOPE AT $X=1$ = $\boxed{0}$ GRAPH IS HORIZONTAL

(d) $f'(2)$ $\boxed{\text{DNE}}$ SHARP POINT ON GRAPH

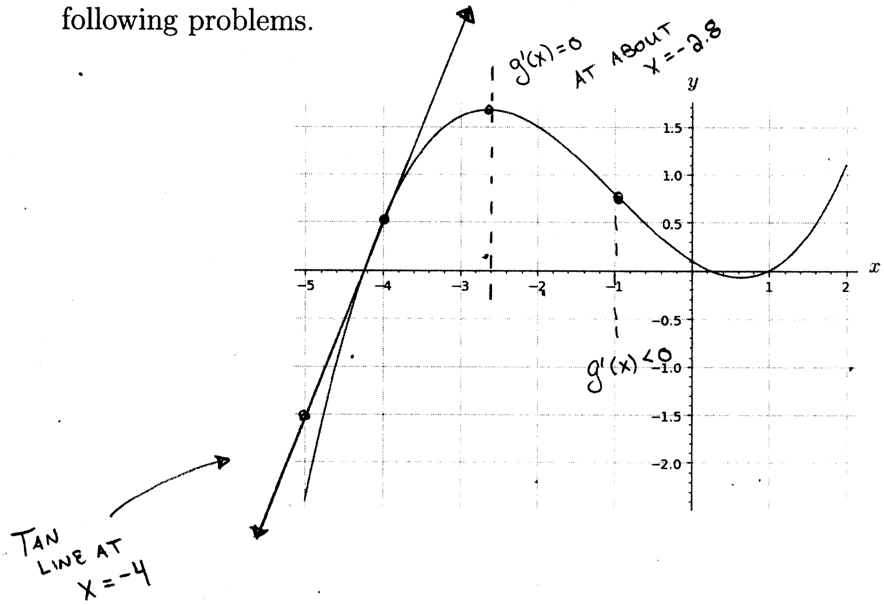
4. (8 points) State two ways in which a function's derivative may fail to exist at a point where the function itself is defined. Then use a single graph to illustrate both ways. Label which is which.

(1) FUNCTION IS NOT DIFFERENTIABLE AT ANY KIND OF DISCONTINUITY

(2) FUNCTION IS NOT DIFFERENTIABLE AT ANY POINT AT WHICH THE GRAPH HAS A SHARP POINT.



5. (7 points) The graph of the function g is shown below. Use the graph to solve the following problems.



(a) Sketch the tangent line at $x = -4$. Then use your tangent line to estimate $g'(-4)$.

SEE ABOVE.

$$g'(-4) \approx \frac{0.5 - (-1.5)}{-4 - (-5)} = \frac{2}{1} = \boxed{2}$$

(b) Find approximate coordinates of a point on the graph at which $g'(x) = 0$. Explain how you know that $g'(x) = 0$ at your point.

$x \approx -2.8$ TANGENT LINE IS HORIZONTAL.
$y \approx 1.6$

(c) Find approximate coordinates of a point on the graph at which $g'(x) < 0$. Explain how you know that $g'(x) < 0$ at your point.

$x = -1$ TANGENT LINE HAS NEGATIVE SLOPE.
$y \approx 0.8$ (FUNCTION IS DECREASING.)

6. (5 points) Use trig identities and the quotient rule to derive our formula for the derivative of $y = \tan x$ from the basic rules for the sine and cosine.

$$\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

7. A rock is thrown vertically upward with a velocity of 48 ft/sec from over the edge of the top of a 160-ft building. Use the position function

$$s(t) = -16t^2 + v_0 t + s_0,$$

where s represents height (in feet) at time t (in seconds), to solve the following problems.

(a) (2 points) Write the function $s(t)$ that gives the rock's height at time t.

$$\boxed{S(t) = -16t^2 + 48t + 160}$$

(b) (2 points) Determine the average rate of change the rock's height over the interval from $t = 0$ to $t = 3$. (Include units with your answer.)

$$\frac{\Delta S}{\Delta t} = \frac{S(3)-S(0)}{3-0} = \frac{[-16(9)+48(3)+160]-160}{3} = \frac{0}{3} = \boxed{0\ \text{FT}/\text{SEC}}$$

(c) (2 points) Determine the function that gives the rock's velocity at time t.

$$\boxed{S'(t) = -32t + 48}$$

(d) (1 point) Determine the rock's velocity after 4 seconds. (Include units with your answer.)

$$S'(4) = -32(4) + 48 = \boxed{-80\ \text{FT}/\text{SEC}}$$

(e) (2 points) What is the acceleration of the rock at time t? (Include units with your answer.)

$$S''(t) = \boxed{-32\ \text{FT}/\text{SEC}^2}$$

(f) (3 points) Determine the rock's maximum height. (Include units with your answer.)

$$S'(t) = 0$$

$$\Rightarrow t = \frac{48}{32} = \frac{3}{2}$$

$$S\left(\tfrac{3}{2}\right) = -16\left(\tfrac{9}{4}\right) + 48\left(\tfrac{3}{2}\right) + 160$$

$$= \boxed{196\ \text{FT}}$$

(g) (3 points) When does the rock hit the ground?

$$S(t) = 0 \Rightarrow -16(t^2 - 3t - 10) = 0$$

$$-16\,(t-5)(t+2) = 0$$

$$\boxed{t = 5\ \text{SEC}}$$

(h) (2 points) What is the speed of the rock when it hits the ground? (Include units with your answer.)

$$|S'(5)| = |-32(5)+48| = |-112| = \boxed{112\ \text{FT}/\text{SEC}}$$

8. (6 points) Determine the higher-order derivative: $\frac{d^3}{dx^3}\underbrace{\left(2x^6 - 5x^4 - 5\sin x\right)}_{f(x)}$

$f'(x) = 12x^5 - 20x^3 - 5\cos x$

$f''(x) = 60x^4 - 60x^2 + 5\sin x$

$\boxed{f'''(x) = 240x^3 - 120x + 5\cos x}$

9. (12 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	0	1	2
$f(x)$	3	-5	0
$f'(x)$	1	-1	-4
$g(x)$	-2	2	1
$g'(x)$	2	6	-7

(a) If $h(x) = 4f(x) - 2g(x)$, compute $h'(2)$.

$h'(x) = 4f'(x) - 2g'(x)$

$h'(2) = 4f'(2) - 2g'(2) = 4(-4) - 2(-7) = -16 + 14 = \boxed{-2}$

(b) If $h(x) = \frac{g(x)}{x^2}$, compute $h'(1)$.

$h'(x) = \frac{g'(x)(x^2) - g(x)(2x)}{x^4}$

$h'(1) = \frac{g'(1)(1^2) - g(1)(2)}{1^4} = \frac{6 - 2(2)}{1} = 6-4 = \boxed{2}$

(c) If $h(x) = f(g(x))$, compute $h'(2)$.

$h'(x) = f'(g(x))\, g'(x)$

$h'(2) = f'(g(2))\, g'(2) = f'(1)\,(-7) = (-1)(-7) = \boxed{7}$

10. (15 points) Differentiate. Do not simplify.

(a) $\frac{d}{dw}\left(\frac{1}{w^2}+w^\pi+\sqrt[5]{w}\right) = \frac{d}{dw}\left(w^{-2}+w^{\pi}+w^{1/5}\right)$

$= \boxed{-2w^{-3}+\pi w^{\pi-1}+\frac{1}{5}w^{-4/5}}$

(b) $\frac{d}{d\theta}(\theta^2\csc\theta) = 2\theta\csc\theta+\theta^2(-\csc\theta\cot\theta)$

$= \boxed{2\theta\csc\theta-\theta^2\csc\theta\cot\theta}$

(c) $\frac{d}{dx}(2\cos^7 x) = 14\cos^6 x\,(-\sin x)$

$= \boxed{-14\cos^6 x\sin x}$

11. (10 points) The function below is discontinuous at two points. Find and classify the discontinuities. For full credit, your work must show how you are using limits and the definition of continuity.

$$f(x)=\begin{cases}\frac{x}{x-3}, & x<3\\ 3x+5, & 3\le x<6\\ x^2-x-5, & x\ge 6\end{cases}$$

DISCONTINUITIES AT

$X=3$ AND $X=6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\underbrace{\frac{x}{x-3}}_{\frac{\text{POS}}{\text{NEG}}=\text{NEG}} \overset{\frac{3}{0}}{=} -\infty$

INFINITE DISCONT AT $X=3$

$\lim_{x\to 6^-} f(x) = 3(6)+5 = 23$

$\lim_{x\to 6^+} f(x) = (6)^2-6-5 = 25$

NOT EQUAL

Jump DISCONT AT $X=6$