Math 131 - Test 3

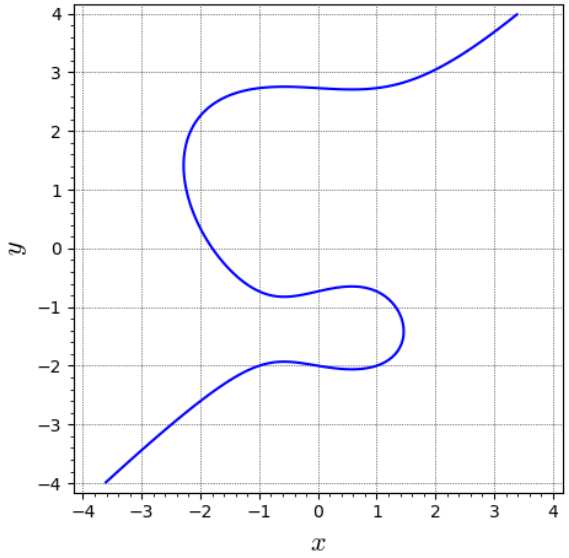
November 6, 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (14 points) The graph of the equation $x^3 - y^3 = x - 6y - 4$ is shown below.



(a) Use implicit differentiation to find a formula for dy/dx.

(b) Use dy/dx to compute the slope of the graph at the point $(1, -2)$. Then determine an equation of the tangent line at $(1, -2)$. (If you could not solve part (a), sketch the tangent line and estimate its slope.)

(c) Find an equation of the line normal to the graph at the point $(1, -2)$. (If you could not solve part (b), sketch the normal line and estimate its slope.)

2. (6 points) Find the instantaneous rate of change of $f(x) = (3x^2 - x - 1)^{13}$ at the point where $x = 1$.

3. (5 points) Let $h(x) = g(f(x))$. Given the following information, compute $h'(5)$.

$$f(5) = 3, \quad f'(5) = -6, \quad g(5) = 9, \quad g'(5) = 0, \quad g(3) = -12, \quad g'(3) = 4$$

4. (4 points) Suppose k is some nonzero constant. Find the derivatives of both $f(x) = \sin(kx)$ and $g(x) = \cos(kx)$.

5. (3 points) Which of the following rules are required to find the derivative of $y = \tan(e^x)$. Circle all that apply.

(a) Product rule (b) Chain rule (c) Power rule

6. (6 points) Suppose f and f^{-1} are differentiable functions. The table below shows the values of $f(x)$ and $f'(x)$ at selected values of x. Find $(f^{-1})'(3)$. Show how you got it.

x	0	1	2	3
$f(x)$	3	8	9	12
$f'(x)$	4	2	1	5

7. (4 points) Let $f(x) = 5x - 2$. Find $(f^{-1})'(7)$.

8. (7 points) Let $g(x) = x^2 \sin^{-1} x$. Find the exact value (not a decimal approximation) of $g'(\sqrt{3}/2)$.

9. (6 points) Find the slope of the line tangent to the graph of $y = \ln(1 + e^{2x})$ at the point where $x = 1$. Round your final answer to the nearest hundredth.

10. (8 points) For $x > 1$, let $y = \dfrac{x^2(x-1)^{3/2}}{\sqrt{x+1}}$. Use logarithmic differentiation to find dy/dx.

11. (6 points) Let $h(x) = \log_7 [(x^2 + 1)^5]$. Compute $h'(2)$. Round your final answer to the nearest hundredth.

12. (6 points) Find the linearization of $f(x) = \dfrac{e^x - e^{-x}}{2}$ at $x = 0$. Then use your linearization to approximate $f(-0.02)$.

13. (6 points) Tests conducted on a vehicle show that its stopping distance, D, when moving at x miles per hour is given by

$$D = 2.5x + 0.5x^2,$$

where D is measured in feet. Use differentials to approximate ΔD when x changes from 25 mph to 26 mph.

14. (6 points) Use a linearization to approximate $(2.99)^3$.

15. (7 points)

(a) Use implicit differentiation to find dy/dx when $xy = 1$.

(b) Solve the equation $xy = 1$ for y so that you have an explicit representation for y. Then find dy/dx.

(c) Show that your answers from part (a) and part (b) are the same.

16. (6 points) Determine each derivative.

(a) $\dfrac{d}{dx}\tan^{-1}(x^5)$

(b) $\dfrac{d}{dt}e^{5t}\cos(2t)$