

November 19, 2025

Score _____

Show all work to receive credit. Supply explanations where necessary. Use extra paper as necessary. This quiz is due December 1.

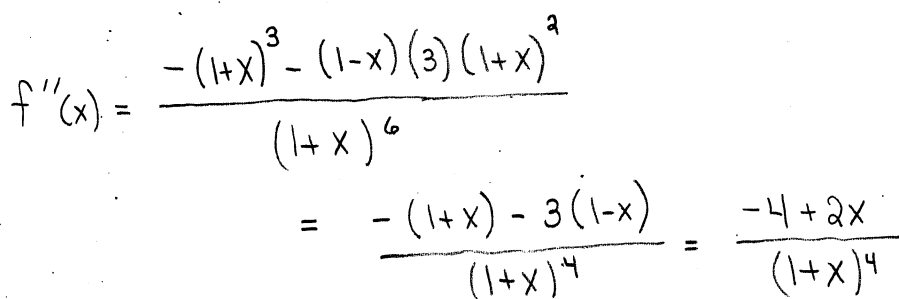
- Part (e) :

H.A.: $y = 0$

$$\lim_{x \rightarrow \pm\infty} \frac{x}{1+2x+x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{0}{0+0+1} = 0$$

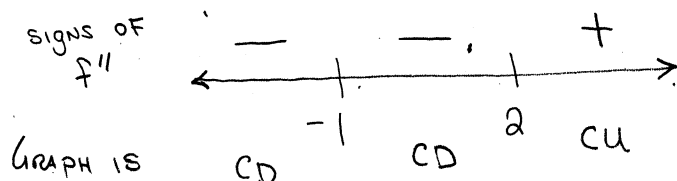
$$f'(x) = 0 \Rightarrow x = 1 \leftarrow \text{CRIT PT.}$$

$$f'(x) \text{ DNE} \Rightarrow x = -1 \leftarrow \text{Not crit pt.}$$



$$f''(x) = 0 \Rightarrow x = 2 \leftarrow \text{PIP}$$

$$f''(x) \text{ DNE} \Rightarrow x = -1 \leftarrow \text{Not a PIP}$$



PARTS (a) & (b) :

f is DECREASING ON $(-\infty, -1) \cup (1, \infty)$

f is increasing on $(-1, 1)$

$$f(1) = \frac{1}{4} \text{ IS A REL MAX.}$$

$X = -1$ is a V.A.

Parents (c) & (d):

Graph of f is CD on $(-\infty, -1) \cup (1, 2)$ and

$$CU \text{ on } (2, \infty).$$

Turn over.

$(2, \frac{2}{9})$ IS AN INFLECTION PT.

$X = -1$ is a V.A.

2. (1 point) Use the 2nd derivative to determine if the graph of $g(x) = x^4 + \sin(1000x)$ is concave up or down at the point where $x = \pi/100$.

$$g'(x) = 4x^3 + 1000 \cos(1000x)$$

$$g''(x) = 12x^2 - 1000^2 \sin(1000x)$$

$$g''\left(\frac{\pi}{100}\right) = \frac{12\pi^2}{100^2} - 1000^2 \sin(10\pi) = \frac{12\pi^2}{100^2} > 0 \Rightarrow$$

GRAPH IS CONCAVE
UP AT $x = \frac{\pi}{100}$

3. (2 points) Evaluate each limit. Show all work.

(a) $\lim_{x \rightarrow \infty} \left(\frac{5 - x^{3/2}}{1 + 2x^{1/2} + 3x^{3/2}} \right) \quad -\infty/\infty$

$$\lim_{x \rightarrow \infty} \left(\frac{5 - x^{3/2}}{1 + 2x^{1/2} + 3x^{3/2}} \right) \cdot \frac{\frac{1}{x^{3/2}}}{\frac{1}{x^{3/2}}} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x^{3/2}} - 1}{\frac{1}{x^{3/2}} + \frac{2}{x} + 3} = \frac{0 - 1}{0 + 0 + 3} = -\frac{1}{3}$$

(b) $\lim_{x \rightarrow -\infty} \left(\frac{5 - x - x^2}{x - 6x^2 - x^3} \right) \quad -\infty/\infty$

$$\lim_{x \rightarrow -\infty} \left(\frac{5 - x - x^2}{x - 6x^2 - x^3} \right) \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow -\infty} \frac{\frac{5}{x^3} - \frac{1}{x^2} - \frac{1}{x}}{\frac{1}{x^2} - \frac{6}{x} - 1} = \frac{0 - 0 - 0}{0 - 0 - 1} = 0$$

4. (2 points) Find the critical numbers of $f(x) = x^4 + 4x^3 - 36x^2$. Then use the 2nd derivative to determine if each gives relative max and relative min.

$$\begin{aligned} f'(x) &= 4x^3 + 12x^2 - 72x \\ &= 4x(x^2 + 3x - 18) \\ &= 4x(x+6)(x-3) \end{aligned}$$

$$f'(x) \text{ DNE NOWHERE.}$$

$$f'(x) = 0 \Rightarrow x=0, x=-6, x=3$$

$$f''(x) = 12x^2 + 24x - 72$$

$$f''(0) = -72 \Rightarrow f(0) = 0 \text{ IS A REL MAX}$$

$$f''(-6) = 216 \Rightarrow f(-6) = -864 \text{ IS A REL MIN}$$

$$f''(3) = 108 \Rightarrow f(3) = -135 \text{ IS A REL MIN}$$