Math 131 - Test 1

September 10, 2025

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (8 points) Use a table of numerical values to approximate the following limit. Your table must show function values at six or more points.

$f(x) = \frac{1-x}{x^2-3x}$

$\lim_{x \to 3} \frac{1-x}{x^2-3x}$ DNE. IT LOOKS LIKE $\lim_{x \to 3^-} f(x) = +\infty$ AND $\lim_{x \to 3^+} f(x) = -\infty$

x	f(x)
2.9	6.55
2.99	66.56
2.999	666.56
2.9999	6666.6

x	f(x)
3.1	-6.77
3.01	-66.78
3.001	-666.8
3.0001	-6666.8

2. (9 points) Given the following information,

$$f(1) = 3, \qquad \lim_{x \to 1} f(x) = -2, \qquad g(1) = 4, \qquad \lim_{x \to 1} g(x) = 12,$$

find the value of each expression below. To receive credit, you must show how you used the limit laws.

(a) $\lim_{x \to 1} (8f(x) - 5g(x))$

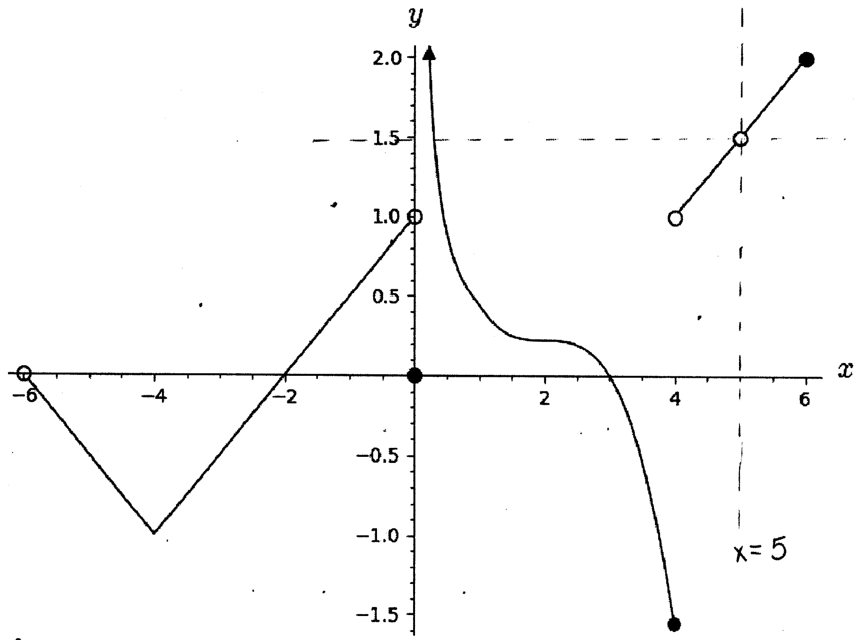
$8 \lim_{x \to 1} f(x) - 5 \lim_{x \to 1} g(x) = 8(-2) - 5(12) = \boxed{-76}$

(b) $\lim_{x \to 1} \frac{f(x)g(x)}{x+1} = \frac{\lim_{x \to 1} f(x) \cdot \lim_{x \to 1} g(x)}{\lim_{x \to 1} (x+1)} = \frac{(-2)(12)}{2} = \boxed{-12}$

(c) $\sqrt{g(1)} + \lim_{x \to 1} (x + f(x))^3 = \sqrt{4} + \left[\lim_{x \to 1} x + \lim_{x \to 1} f(x)\right]^3$

$= 2 + [1 + (-2)]^3 = 2 + -1 = \boxed{1}$

3. (12 points) Referring to the graph of $y = f(x)$ shown below, estimate each of the following or explain why it does not exist.



(a) $\lim_{x \to -4} f(x)$ = $\boxed{-1}$

(b) $\lim_{x \to 0^+} f(x)$ = $\boxed{+\infty}$

(c) $\lim_{x \to 0^-} f(x)$ = $\boxed{1}$

(d) $\lim_{x \to 5} f(x)$ = $\boxed{1.5}$

(e) $\lim_{x \to 4} f(x)$ $\boxed{\text{DNE}}$ BECAUSE $\underbrace{\lim_{x \to 4^-} f(x) \approx -1.5 \text{ AND } \lim_{x \to 4^+} f(x) \approx 1}_{\text{FAILURE \#1}}$

(f) $\lim_{x \to 6} f(x)$ $\boxed{\text{DNE}}$ BECAUSE f IS $\underbrace{\text{NOT DEFINED FOR } X > 6}_{\text{FAILURE \#4}}$

4. (7 points) The function $y = h(x)$ is defined below.

$$h(x) = \begin{cases} |4x| + \sin(\pi x), & x < 2 \\ 3x^2 - 5x + 1, & x \geq 2 \end{cases}$$

Find each limit analytically. If the limit does not exist, you must say why.

(a) $\lim_{x \to 1} h(x)$

$= \lim_{x \to 1} \left[|4x| + \sin(\pi x) \right] = 4 + \sin \pi = \boxed{4}$

(b) $\lim_{x \to 5} h(x) = \lim_{x \to 5} \left(3x^2 - 5x + 1 \right) = 3(25) - 5(5) + 1 = \boxed{51}$

(c) $\lim_{x \to 2} h(x)$

$\lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} \left[|4x| + \sin(\pi x) \right] = 8 + \sin 2\pi = 8$

$\lim_{x \to 2^+} h(x) = \lim_{x \to 2^+} \left[3x^2 - 5x + 1 \right] = 12 - 10 + 1 = 3$

LIMIT DNE

SIDED LIMITS NOT EQUAL.

5. (10 points) These limits DO NOT EXIST. Carefully explain why each limit fails to exist. Show work that supports your answer.

(a) $\lim_{x \to 0} \frac{\sqrt[3]{x}}{x}$

% MORE WORK

(IN FACT, $\lim_{x \to 0} \frac{\sqrt[3]{x}}{x} = +\infty$)

$\lim_{x \to 0} \frac{x^{1/3}}{x} = \lim_{x \to 0} \frac{1}{x^{2/3}}$

$1/0$

THE $\frac{1}{0}$ FORM INDICATES THAT THE LIMIT DNE BECAUSE OF UNBOUNDED GROWTH.

(b) $\lim_{x \to 9} \frac{x^2 - 9x}{|x - 9|}$

%

$\lim_{x \to 9^-} \frac{x(x-9)}{|x-9|} = \lim_{x \to 9^-} \frac{x(\cancel{x-9})}{-(\cancel{x-9})} = -9$

$\lim_{x \to 9^+} \frac{x(x-9)}{|x-9|} = \lim_{x \to 9^+} \frac{x(\cancel{x-9})}{\cancel{x-9}} = 9$

LIMIT FROM LEFT $\neq$ LIMIT FROM RIGHT

(c) $\lim_{x \to 0} g(x)$, where $g(x) = \begin{cases} x + 2, & 0 \leq x < 1 \\ 5x + 7, & x \geq 1 \end{cases}$

THIS IS A TWO-SIDED LIMIT, BUT $g(x)$ IS NOT DEFINED TO THE LEFT OF $X = 0$

$\lim_{x \to 0^-} g(x)$ DNE, $\lim_{x \to 0^+} g(x) = 2$

6. (24 points) **Determine each limit analytically**, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE. You will not be given credit if you get your answer from a table of values or a graph.

(a) $\lim_{x \to 1} \frac{x^2+3x-4}{x^2-1}$ $0/0$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1} \frac{(x+4)}{(x+1)} = \boxed{\frac{5}{2}}$$

(b) $\lim_{u \to 0} \frac{(u-2)^2-4}{u}$ $0/0$

$$= \lim_{u\to 0} \frac{u^2-4u+4-4}{u} = \lim_{u\to 0} \frac{u^2-4u}{u} = \lim_{u\to 0} \frac{\cancel{u}(u-4)}{\cancel{u}}$$

$$= \lim_{u\to 0} (u-4) = \boxed{-4}$$

(c) $\lim_{x \to 0} \frac{(x+3)\sin 2x}{4x}$ $0/0$ $= \lim_{x\to 0} \left(\frac{x+3}{2} \cdot \frac{\sin 2x}{2x} \right)$

$$= \underbrace{\lim_{x\to 0} \frac{x+3}{2}}_{3/2} \cdot \underbrace{\lim_{x\to 0} \frac{\sin 2x}{2x}}_{1} = \frac{3}{2} \cdot 1 = \boxed{\frac{3}{2}}$$

(d) $\lim_{t \to 9} \frac{3-\sqrt{t}}{18-2t}$ $0/0$

$$\lim_{t\to 9} \frac{3-\sqrt{t}}{2(9-t)} \cdot \frac{3+\sqrt{t}}{3+\sqrt{t}} = \lim_{t\to 9} \frac{9-t}{2\cancel{(9-t)}(3+\sqrt{t})}$$

$$= \lim_{t\to 9} \frac{1}{2(3+\sqrt{t})} = \frac{1}{2(3+3)} = \boxed{\frac{1}{12}}$$

7. (4 points) Suppose $f(x)$ is a function for which

$$3x \le f(x) \le x^3 + 2$$

whenever $0 < x < 2$. Compute $\lim_{x\to 1} f(x)$ and explain your reasoning.

$\lim_{X\to 1} 3X = 3 = \lim_{X\to} (X^3 + 2)$ $\therefore$ $\boxed{\lim_{X\to 1} f(X) = 3 \text{ BY SQUEEZE THM.}}$

8. (12 points) For each part of this problem, **determine analytically** whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x\to 5} \frac{\sqrt{x}}{|x-5|}$

5/0 UNBOUNDED

CHECK BOTH SIDES OF $X = 5$...

$\boxed{\lim_{X\to 5} \frac{\sqrt{5}}{|X-5|} = +\infty}$

FROM LEFT OF $X=5$

$\frac{\sqrt{X}}{|X-5|} = \frac{\text{POS}}{\text{POS}} = \text{POS}$

FROM RIGHT OF $X=5$

$\frac{\sqrt{X}}{|X-5|} = \frac{\text{POS}}{\text{POS}} = \text{POS}$

(b) $\lim_{x\to 2^-} \left(\frac{7x}{2-x}\right)$

14/0 UNBOUNDED

CHECK LEFT OF $X = 2$...

LEFT OF $X=2$

$\frac{7X}{2-X} = \frac{\text{POS}}{\text{POS}} = \text{POS}$

$\boxed{\lim_{X\to 2^-} \frac{7X}{2-X} = +\infty}$

(c) $\lim_{x\to \pi^+} \left(\frac{3}{\sin x}\right)$

3/0 UNBOUNDED

CHECK RIGHT OF $X = \pi$...

RIGHT OF $X = \pi$

$\frac{3}{\text{SIN } X} = \frac{\text{POS}}{\text{NEG}} = \text{NEG}$

$\boxed{\lim_{X\to \pi^+} \frac{3}{\text{SIN } X} = -\infty}$

(d) $\lim_{x\to 1} \left[\frac{x-5}{x-1}\right]$

-4/0 UNBOUNDED

CHECK BOTH SIDES OF $X = 1$...

LEFT OF $X=1$

$\frac{X-5}{X-1} = \frac{\text{NEG}}{\text{NEG}} = \text{POS}$

RIGHT OF $X=1$

$\frac{X-5}{X-1} = \frac{\text{NEG}}{\text{POS}} = \text{NEG}$

$\boxed{\lim_{X\to 1^-} \frac{X-5}{X-1} = +\infty \qquad \lim_{X\to 1^+} \frac{X-5}{X-1} = -\infty}$

LIMIT DNE.

9. (4 points) Determine all vertical asymptotes of the graph of $R(x) = \frac{x^2+2x}{x^3-4x}$.

$R(X) = \frac{X(X+2)}{X(X^2-4)} = \frac{X(X+2)}{X(X+2)(X-2)} = \frac{1}{X-2}$; $X \neq 0, -2$

$\boxed{\text{THE ONLY V.A. IS } X = 2.}$

10. (2 points) When evaluating the limit of a rational function, you tried direct substitution and obtained a nonzero over zero form. Which of these must be true?

(a) You must use L'Hôpital's rule to determine the limit.

(b) The limit does not exist because the function values grow without bound around the limit point.

(c) The limit exists, but could be any number.

(d) No conclusion can be drawn from that form.

11. (2 points) The function f is defined for all real numbers, and $f(2) = 5$. Which one of these statements must be true?

(a) $\lim_{x \to 2} f(x)$ exists.

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 2} f(x) = 5$

(d) Nothing can be said about $\lim_{x \to 2} f(x)$ without more information.

12. (2 points) Suppose the graph of f has a vertical asymptote at $x = -2$. Which of these cannot be true?

(a) $f(-2) = 13$

(b) $\lim_{x \to -2} f(x) = 1$

(c) $\lim_{x \to -2^+} f(x) = 7$

(d) $\lim_{x \to -2} f(x) = -\infty$

13. (2 points) f is a polynomial function, and you would like to find $\lim_{x \to 0} f(x)$. Which one of these is false?

(a) You can find the limit by direct substitution.

(b) It is impossible to obtain a $0/0$ form by direct substitution.

(c) $\lim_{x \to 0} f(x) = f(0)$

(d) f is not defined on an interval around $x = 0$.

14. (2 points) Which one of these is a possible description of a rational function?

(a) A polynomial divided by another polynomial.

(b) A polynomial divided by a trigonometric function.

(c) A product of an exponential function and a polynomial.

(d) A quotient of a radical function and an exponential function.