# Math 131 - Test 3

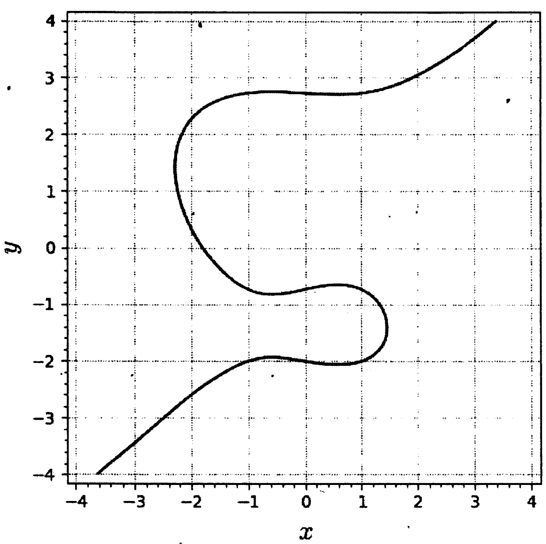
November 5, 2025

Name key

Score ________

**Show all work to receive full credit**. Supply explanations where necessary.

1. (14 points) The graph of the equation $x^3 - y^3 = x - 6y - 4$ is shown below.




(a) Use implicit differentiation to find a formula for $dy/dx$.

$$\frac{d}{dx}(x^3 - y^3) = \frac{d}{dx}(x - 6y - 4)$$

$$3x^2 - 3y^2\frac{dy}{dx} = 1 - 6\frac{dy}{dx}$$

$$(6 - 3y^2)\frac{dy}{dx} = 1 - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2}{6 - 3y^2}}$$

(b) Use $dy/dx$ to compute the slope of the graph at the point $(1, -2)$. Then determine an equation of the tangent line at $(1, -2)$. (If you could not solve part (a), sketch the tangent line and estimate its slope.)

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,-2)} = \frac{1-3}{6-12} = \frac{-2}{-6} = \boxed{\frac{1}{3}}$$

$$\boxed{y + 2 = \frac{1}{3}(x - 1)}$$

or

$$y = \frac{1}{3}x - \frac{7}{3}$$

(c) Find an equation of the line normal to the graph at the point $(1, -2)$. (If you could not solve part (b), sketch the normal line and estimate its slope.)

$$m_\perp = -\frac{3}{1} = -3$$

$$\boxed{y + 2 = -3(x - 1)}$$

OR

$$y = -3x + 1$$

2. (6 points) Find the instantaneous rate of change of $f(x) = (3x^2 - x - 1)^{13}$ at the point where $x = 1$.

$$f'(x) = 13(3x^2 - x - 1)^{12}(6x - 1)$$

$$f'(1) = 13(1)^{12} \cdot (5) = \boxed{65}$$

3. (5 points) Let $h(x) = g(f(x))$. Given the following information, compute $h'(5)$.

$$f(5) = 3, \quad f'(5) = -6, \quad g(5) = 9, \quad g'(5) = 0, \quad g(3) = -12, \quad g'(3) = 4$$

$$h'(x) = g'(f(x))f'(x)$$

$$h'(5) = g'(f(5))f'(5)$$

$$= g'(3)(-6) = 4(-6) = \boxed{-24}$$

4. (4 points) Suppose $k$ is some nonzero constant. Find the derivatives of both $f(x) = \sin(kx)$ and $g(x) = \cos(kx)$.

$$f'(x) = \boxed{\frac{d}{dx}\sin(kx) = k\cos(kx)}$$

$$g'(x) = \boxed{\frac{d}{dx}\cos(kx) = -k\sin(kx)}$$

5. (3 points) Which of the following rules are required to find the derivative of $y = \tan(e^x)$. Circle all that apply.

(a) Product rule (b) Chain rule (c) Power rule

6. (6 points) Suppose $f$ and $f^{-1}$ are differentiable functions. The table below shows the values of $f(x)$ and $f'(x)$ at selected values of $x$. Find $(f^{-1})'(3)$. Show how you got it.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 3 | 8 | 9 | 12 |
| $f'(x)$ | 4 | 2 | 1 | 5 |

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(0)} = \boxed{\frac{1}{4}}$$

$f^{-1}(3) = 0$

BECAUSE $f(0) = 3$

7. (4 points) Let $f(x) = 5x - 2$. Find $(f^{-1})'(7)$.

$$(f^{-1})'(7) = \frac{1}{f'(f^{-1}(7))} = \frac{1}{f'(\frac{9}{5})} = \boxed{\frac{1}{5}}$$

$f'(x) = 5$

$f^{-1}(7) = X \Leftrightarrow 5X - 2 = 7 \Leftrightarrow X = \frac{9}{5}$ ← THIS DOESN'T MATTER SINCE $f'(x)$ IS A CONSTANT FUNC.

8. (7 points) Let $g(x) = x^2 \sin^{-1} x$. Find the exact value (not a decimal approximation) of $g'(\sqrt{3}/2)$.

$$g'(x) = 2x \sin^{-1} x + x^2 \left( \frac{1}{\sqrt{1-x^2}} \right)$$

$$g'\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3} \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{3}{4} \left( \frac{1}{\sqrt{1 - \frac{3}{4}}} \right)$$

$$= \frac{\sqrt{3}\,\pi}{3} + \frac{3/4}{1/2} = \boxed{\frac{\sqrt{3}\,\pi}{3} + \frac{3}{2}}$$

9. (6 points) Find the slope of the line tangent to the graph of $y = \ln(1 + e^{2x})$ at the point where $x = 1$. Round your final answer to the nearest hundredth.

$$\frac{dy}{dx} = \frac{1}{1+e^{2x}}\left(2e^{2x}\right) = \frac{2e^{2x}}{1+e^{2x}}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \boxed{\frac{2e^2}{1+e^2} \approx 1.76}$$

10. (8 points) For $x > 1$, let $y = \dfrac{x^2(x-1)^{3/2}}{\sqrt{x+1}}$. Use logarithmic differentiation to find $dy/dx$.

$$\ln y = 2\ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{3}{2}\,\frac{1}{x-1} - \frac{1}{2}\,\frac{1}{x+1}$$

$$\boxed{\frac{dy}{dx} = \left(\frac{2}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right)\left(\frac{x^2(x-1)^{3/2}}{\sqrt{x+1}}\right)}$$

11. (6 points) Let $h(x) = \log_7[(x^2+1)^5]$. Compute $h'(2)$. Round your final answer to the nearest hundredth.

$$h(x) = \frac{5}{\ln 7}\ln(x^2+1)$$

$$h'(x) = \frac{5}{\ln 7}\,\frac{2x}{x^2+1} \qquad h'(2) = \frac{20}{(\ln 7)(5)} = \frac{4}{\ln 7}$$

$$\approx \boxed{2.06}$$

12. (6 points) Find the linearization of $f(x) = \dfrac{e^x - e^{-x}}{2}$ at $x = 0$. Then use your linearization to approximate $f(-0.02)$.

$f(0) = \dfrac{1-1}{2} = 0$

$f'(x) = \dfrac{e^x + e^{-x}}{2}$

$f'(0) = \dfrac{1+1}{2} = 1$

$L(x) = 0 + 1(x-0)$

$\boxed{L(x) = x}$

$\boxed{f(-0.02) \approx L(-0.02) = -0.02}$

13. (6 points) Tests conducted on a vehicle show that its stopping distance, $D$, when moving at $x$ miles per hour is given by

$$D = 2.5x + 0.5x^2,$$

where $D$ is measured in feet. Use differentials to approximate $\Delta D$ when $x$ changes from 25 mph to 26 mph.

$dD = (2.5 + x)\, dx$

$\Delta D \approx (2.5 + x)\, \Delta x$

$x = 25$

$\Delta x = 1$

$\Rightarrow \quad \Delta D \approx (2.5 + 25)(1) = \boxed{27.5 \text{ FT}}$

14. (6 points) Use a linearization to approximate $(2.99)^3$.

LINEARIZE $f(x) = x^3$ AT $x = 3$.

$f(3) = 27$

$f'(3) = 3x^2$

$f'(3) = 27$

$\boxed{L(x) = 27 + 27(x-3)}$

$(2.99^3) \approx 27 + 27(-0.01)$

$= \boxed{26.73}$

15. (7 points)

(a) Use implicit differentiation to find $dy/dx$ when $xy = 1$.

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x}}$$

(b) Solve the equation $xy = 1$ for $y$ so that you have an explicit representation for $y$. Then find $dy/dx$.

$$y = \frac{1}{x} = x^{-1}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{x^2}}$$

(c) Show that your answers from part (a) and part (b) are the same.

$$\text{PART (b)} \rightarrow \frac{dy}{dx} = \underbrace{\frac{-1}{x^2} = \frac{-xy}{x^2}}_{xy=1} = \frac{-y}{x} = \frac{dy}{dx} \leftarrow \text{PART (a)}$$

16. (6 points) Determine each derivative.

(a) $\dfrac{d}{dx}\tan^{-1}(x^5)$

$$= \frac{1}{(x^5)^2 + 1}(5x^4) = \boxed{\frac{5x^4}{x^{10}+1}}$$

(b) $\dfrac{d}{dt}e^{5t}\cos(2t)$

$$= \boxed{5e^{5t}\cos(2t) - 2e^{5t}\sin(2t)}$$