Math 131 - Final Exam
Fall 2025

Name key
Score

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit.

   (a) $\lim\limits_{x\to 0} \dfrac{(x-2)^2-4}{x}$ $0/0$ $= \lim\limits_{x\to 0} \dfrac{x^2-4x+4-4}{x} = \lim\limits_{x\to 0} \dfrac{x(x-4)}{x}$

   $= \lim\limits_{x\to 0} (x-4) = \boxed{-4}$

   (b) $\lim\limits_{t\to 16} \dfrac{\sqrt{t}-4}{t-16}$ $0/0$ $\cdot \dfrac{\sqrt{t}+4}{\sqrt{t}+4} = \lim\limits_{t\to 16} \dfrac{t-16}{(t-16)(\sqrt{t}+4)} = \lim\limits_{t\to 16} \dfrac{1}{\sqrt{t}+4} = \boxed{\dfrac{1}{8}}$

2. (10 points) The function g has exactly one discontinuity.

$$g(x) = \begin{cases} \dfrac{\sin x}{x}, & x < 0 \\ x^2+1, & 0 \le x \le 2 \\ \ln(x-2), & x > 2 \end{cases}$$

   (a) Show that g is continuous at $x = 0$.

   $\lim\limits_{x\to 0^-} g(x) = \lim\limits_{x\to 0^-} \dfrac{\sin x}{x} = 1$

   $\lim\limits_{x\to 0^+} g(x) = \lim\limits_{x\to 0^+} (x^2+1) = 1$

   $g(0) = 1 \Rightarrow \lim\limits_{x\to 0} g(x) = g(0)$

   (b) Find and classify the single point of discontinuity.

   Since

   $\lim\limits_{x\to 2^+} g(x) = \lim\limits_{x\to 2^+} \ln(x-2) = -\infty$

   $x = 2$ IS AN INFINITE DISCONTINUITY.

3. (10 points) An object is launched vertically upward from over the edge of a building. The object's height (in meters) after t seconds is given by

$$s(t) = -4.9t^2 + 14.7t + 49.$$

(a) What is the object's the maximum height? Give units with your answer.

$s'(t) = -9.8t + 14.7$

$s'(t) = 0 \Rightarrow t = \frac{14.7}{9.8} = \frac{3}{2}$ sec

$s(\frac{3}{2}) = \boxed{60.025 \text{ m}}$

(b) What is object's speed when it hits the ground? Give units with your answer.

$s(t) = 0 \Rightarrow -4.9(t^2 - 3t - 10) = 0$

$-4.9(t-5)(t+2) = 0$

$t = 5$ sec

$|s'(5)| = |-34.3|$

$= \boxed{34.3 \text{ m/s}}$

4. (10 points) Let $f(x) = \dfrac{x^2 - 3}{x - 2}$. Find open intervals on which f is increasing/decreasing and identify all relative extreme values.

$$f'(x) = \frac{2x(x-2) - (x^2-3)}{(x-2)^2} = \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2}$$

$f'(x)$ DNE $\Rightarrow x = 2$

$f'(x) = 0 \Rightarrow x = 3, x = 1$

SIGNS OF f': + | − | − | +

f IS: INC | $x=1$ CRIT | DEC | $x=2$ V.A. | DEC | $x=3$ CRIT | INC

$\cap$ $\cup$

$\boxed{\begin{array}{l} f \text{ IS INCREASING ON } (-\infty, 1) \cup (3, \infty) \\ f \text{ IS DECREASING ON } (1, 2) \cup (2, 3) \\ f(1) = 2 \text{ IS A REL MAX}, \quad f(3) = 6 \text{ IS A REL MIN} \end{array}}$

5. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\displaystyle\lim_{n\to\infty} \frac{n - n^2 - 5n^4}{5n^2 + 7n^3 - 2n^4}$ $\quad -\infty/-\infty$

$$\lim_{n\to\infty} \frac{n-n^2-5n^4}{5n^2+7n^3-2n^4} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^3} - \frac{1}{n^2} - 5}{\frac{5}{n^2} + \frac{7}{n} - 2}$$

$$= \frac{0-0-5}{0+0-2} = \boxed{\frac{5}{2}}$$

$-\infty/-\infty$

(b) $\displaystyle\lim_{x\to 0+} \frac{\ln(e^x - 1)}{\ln x}$

$$= \lim_{x\to 0^+} \frac{\frac{e^x}{e^x-1}}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{xe^x}{e^x-1} \overset{0/0}{=} \lim_{x\to 0^+} \frac{e^x - xe^x}{e^x} = \frac{1-0}{1}$$

$$= \boxed{1}$$

6. (10 points) Let $g(x) = x^4 - 6x^3 - 60x^2 + 12x + 1$. Find open intervals on which the graph of g is concave up/down and identify all inflection points.

$g'(x) = 4x^3 - 18x^2 - 120x + 12$

$g''(x) = 12x^2 - 36x - 120$

$g''(x) = 12(x^2 - 3x - 10)$

$= 12(x-5)(x+2) = 0$

$\Rightarrow x = 5, \ x = -2$

SIGNS OF g'': $+$ $-$ $+$

$x=-2$ $x=5$

GRAPH IS: CU   CD   CU

INF.   INF.

GRAPH IS CONCAVE UP ON $(-\infty,-2) \cup (5,\infty)$ AND CONCAVE DOWN ON $(-2,5)$

$(-2,-199)$, $(5,-1564)$ ARE INFLECTION PTS.

7. (10 points) You do not know the function $y = f(x)$, but you do know the following:

$$\frac{dy}{dx} = 8 + \frac{2y}{12+3x} \qquad \text{and} \qquad \underbrace{y = 3 \text{ when } x = 0}_{f(0)=3}.$$

(a) Find the linearization of $y = f(x)$ at $x = 0$.

$f(0)=3$

$$f'(0) = \left.\frac{dy}{dx}\right|_{x=0} = 8 + \frac{2(3)}{12} = 8 + \frac{1}{2} = \frac{17}{2}$$

$$L(x) = 3 + \frac{17}{2}(x-0) \Rightarrow \boxed{L(x) = 3 + \frac{17}{2}x}$$

(b) Use your linearization to approximate y when $x = 0.2$.

$$f(0.2) \approx L(0.2) = 3 + \frac{17}{2}(0.2)$$

$$= 3 + 1.7 = \boxed{4.7}$$

(c) Explain why your linearization would not be useful for approximating $f(2)$.

$X = 2$ IS PROBABLY TOO FAR FROM $X = 0$ FOR OUR LINEARIZATION TO BE USEFUL.

8. (10 points) Evaluate each indefinite integral. (Be sure to check your answer by differentiation.)

(a) $\int \left(4x^2 + \frac{1}{x} + x^{1/3}\right) dx$

$$= \boxed{\frac{4}{3}x^3 + \ln|x| + \frac{3}{4}x^{4/3} + C}$$

(b) $\int (e^{2x} + \sin x)\, dx$

$$= \boxed{\frac{1}{2}e^{2x} - \cos x + C}$$

9. (10 points) Left $f(x) = \sqrt{x}, \quad 1 \le x \le 4$.

(a) Use 6 subintervals of equal length and subinterval midpoints to compute the corresponding Riemann sum for f over $[1, 4]$.

$\Delta X = \frac{4-1}{6} = 0.5$

PARTITION:

$1 < 1.5 < 2 < 2.5 < 3 < 3.5 < 4$

$C_1 = 1.25 \qquad C_4 = 2.75$

$C_2 = 1.75 \qquad C_5 = 3.25$

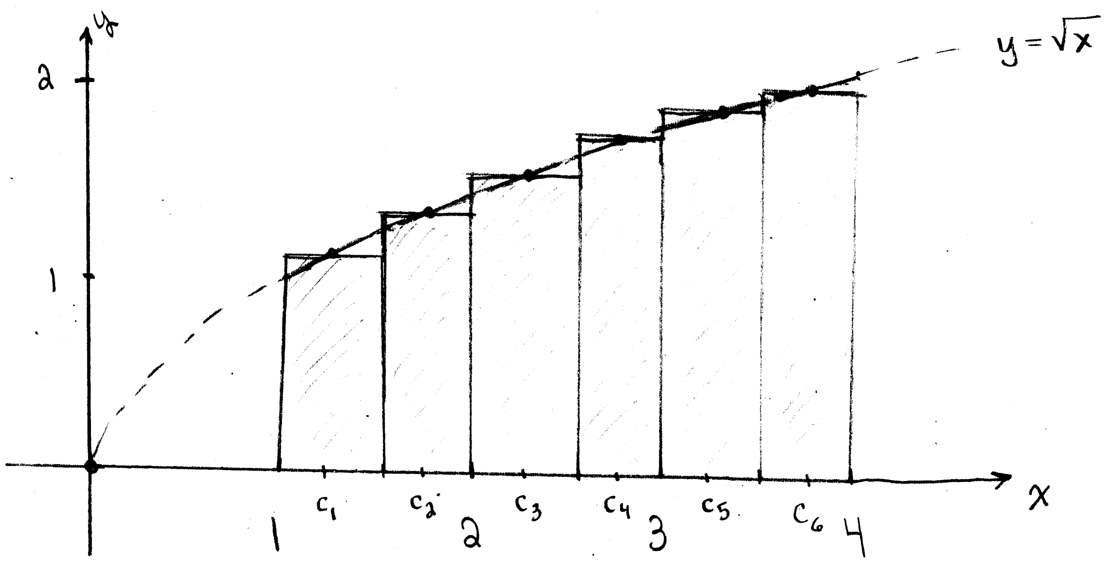
$C_3 = 2.25 \qquad C_6 = 3.75$

RIEMANN SUM =

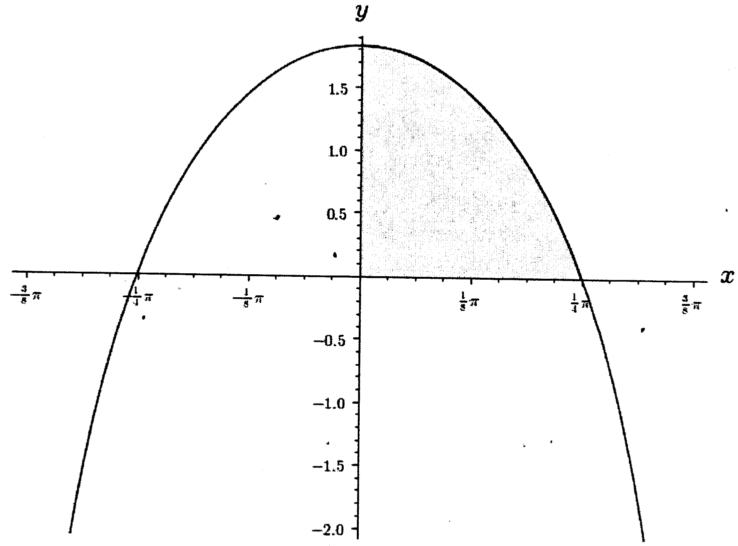
$0.5(\sqrt{1.25} + \sqrt{1.75} + \sqrt{2.25} + \sqrt{2.75} + \sqrt{3.25} + \sqrt{3.75})$

≈ 4.669

(b) Sketch the graph of f over the interval $[1, 4]$ and then sketch (in detail) the rectangles associated with your Riemann sum.



10. (10 points) The graph of $y = 2\sqrt{2}\cos x - \sec^2 x$ is shown below.



(a) Write the definite integral that gives the area of the shaded region between $x = 0$ and $x = \pi/4$.

$$\int_0^{\pi/4} \left(2\sqrt{2}\cos x - \sec^2 x\right) dx$$

(b) Use the Fundamental Theorem of Calculus to evaluate your definite integral.

$$\int_0^{\pi/4} \left(2\sqrt{2}\cos x - \sec^2 x\right) dx = 2\sqrt{2}\sin x - \tan x \Big|_0^{\pi/4}$$

$$= \left(2\sqrt{2}\sin\frac{\pi}{4} - \tan\frac{\pi}{4}\right) - \left(2\sqrt{2}\sin 0 - \tan 0\right)$$

$$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) - 1 - 0 + 0$$

$$= 2 - 1 = \boxed{1}$$