Math 131 - Final Exam

Fall 2025

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to determine each limit.

   (a) $\lim_{x \to 0} \dfrac{(x-2)^2 - 4}{x}$

   (b) $\lim_{t \to 16} \dfrac{\sqrt{t} - 4}{t - 16}$

2. (10 points) The function g has exactly one discontinuity.

$$g(x) = \begin{cases} \dfrac{\sin x}{x}, & x < 0 \\ x^2 + 1, & 0 \leq x \leq 2 \\ \ln(x-2), & x > 2 \end{cases}$$

   (a) Show that g is continuous at $x = 0$.

   (b) Find and classify the single point of discontinuity.

3. (10 points) An object is launched vertically upward from over the edge of a building. The object's height (in meters) after t seconds is given by

$$s(t) = -4.9t^2 + 14.7t + 49.$$

(a) What is the object's the maximum height? Give units with your answer.

(b) What is object's speed when it hits the ground? Give units with your answer.

4. (10 points) Let $f(x) = \dfrac{x^2 - 3}{x - 2}$. Find open intervals on which f is increasing/decreasing and identify all relative extreme values.

5. (10 points) Use any analytical method (not a table or graph) to determine each limit.

(a) $\lim_{n\to\infty} \dfrac{n - n^2 - 5n^4}{5n^2 + 7n^3 - 2n^4}$

(b) $\lim_{x\to 0^+} \dfrac{\ln(e^x - 1)}{\ln x}$

6. (10 points) Let $g(x) = x^4 - 6x^3 - 60x^2 + 12x + 1$. Find open intervals on which the graph of g is concave up/down and identify all inflection points.

7. (10 points) You do not know the function $y = f(x)$, but you do know the following:

$$\frac{dy}{dx} = 8 + \frac{2y}{12 + 3x} \qquad \text{and} \qquad y = 3 \text{ when } x = 0.$$

(a) Find the linearization of $y = f(x)$ at $x = 0$.

(b) Use your linearization to approximate y when $x = 0.2$.

(c) Explain why your linearization would not be useful for approximating $f(2)$.

8. (10 points) Evaluate each indefinite integral. (Be sure to check your answer by differentiation.)

(a) $\displaystyle\int \left(4x^2 + \frac{1}{x} + x^{1/3}\right) dx$

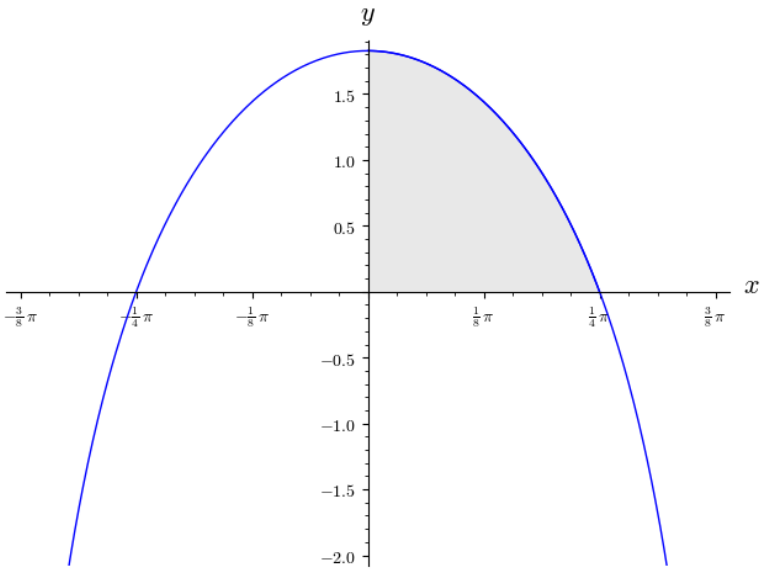
(b) $\displaystyle\int (e^{2x} + \sin x)\, dx$

9. (10 points) Left $f(x) = \sqrt{x}, \quad 1 \leq x \leq 4$.

(a) Use 6 subintervals of equal length and subinterval midpoints to compute the corresponding Riemann sum for f over $[1, 4]$.

(b) Sketch the graph of f over the interval $[1, 4]$ and then sketch (in detail) the rectangles associated with your Riemann sum.

10. (10 points) The graph of $y = 2\sqrt{2}\cos x - \sec^2 x$ is shown below.



(a) Write the definite integral that gives the area of the shaded region between $x = 0$ and $x = \pi/4$.

(b) Use the Fundamental Theorem of Calculus to evaluate your definite integral.