Math 131 - Quiz 3 (IC)

February 27, 2020

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Use the appropriate differentiation rules to determine the derivative of each function.

   (a) $f(x) = 4x^3 - 7x^2 + x - 9$

   $$f'(x) = 12x^2 - 14x + 1$$

   (b) $y = \dfrac{1}{t} + t^{3/5} + \sqrt{t} \;=\; t^{-1} + t^{3/5} + t^{1/2}$

   $$\frac{dy}{dt} = -t^{-2} + \frac{3}{5}t^{-2/5} + \frac{1}{2}t^{-1/2}$$

   $$= -\frac{1}{t^2} + \frac{3}{5\sqrt[5]{t^2}} + \frac{1}{2\sqrt{t}}$$

2. (2 points) Find an equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at the point where $x = 4$.

   $$f'(x) = \frac{1}{2}x^{-1/2}$$

   $$\text{Slope} = f'(4) = \frac{1}{2}(4)^{-1/2} = \frac{1}{4}$$

   Point : $x = 4$, $y = f(4) = 2$

   $$(4, 2)$$

   Tangent Line : $y - 2 = \frac{1}{4}(x - 4)$

   or $y = \frac{1}{4}x + 1$

Show all work to receive full credit. Supply explanations when necessary. This quiz is due no later than 3:15pm on March 3.

1. (2 points) The following table gives information about the functions f and g.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-7	0	4	2
1	-8	-1	7	5
2	-7	4	DNE	DNE
3	2	15	1	5

Use this information to find an equation of the line tangent to the graph of $y = f(x)g(x)$ at the point where $x = 1$.

$$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

$$m = \frac{dy}{dx}\Big|_{x=1} = f'(1)g(1) + f(1)g'(1)$$

$$= (-1)(7) + (-8)(5)$$

$$= -47$$

POINT: $x = 1$

$$y = f(1)g(1) =$$
$$(-8)(7) = -56$$

TAN LINE :

$$y + 56 = -47(x-1)$$

OR

$$y = -47x - 9$$

2. (2 points) An object is thrown straight up from over the side of a 110-ft building. If the object's initial velocity is 50 ft/sec, how high will it go and when will it land on the ground.

$$h(t) = -16t^2 + 50t + 110$$

MAX HEIGHT
WHEN. $h'(t) = -32t + 50 = 0$

$$\Rightarrow t = \frac{50}{32} = 1.5625 \text{ sec}$$

$$h(1.5625) = 149.0625 \text{ FT}$$

$$\underbrace{}_{\text{MAX HEIGHT}}$$

HITS GROUND WHEN

$$h(t) = 0 \dots$$

$$-16t^2 + 50t + 110 = 0$$

$$t = \frac{-50 - \sqrt{50^2 + 4(16)(110)}}{-32}$$

$$\approx 4.615 \text{ sec}$$

3. (1 point) Find $f''(x)$ if $f(x) = 8 \sec x$.

$$f'(x) = 8 \sec x \tan x$$

$$f''(x) = 8 \sec x (\sec^2 x) + 8(\sec x \tan x) \tan x$$

$$= 8 \sec^3 x + 8 \sec x \tan^2 x$$

4. (1 point) Find the instantaneous rate of change of $g(x) = \dfrac{\sqrt{x}+6}{x^2-5x}$ at the point $(4, -2)$.

$$g'(x) = \frac{(x^2-5x)(\frac{1}{2}x^{-1/2}) - (\sqrt{x}+6)(2x-5)}{(x^2-5x)^2}$$

$$g'(4) = \frac{(-4)(\frac{1}{4}) - (8)(3)}{(-4)^2} = \frac{-25}{16}$$