Math 131 - Quiz 4

April 2, 2020

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than April 7.

---

1. (3 points) Consider the equation $x^2 + 2xy - 3y^2 = -7$ and its graph.

   (a) Use implicit differentiation to find dy/dx.

$$\frac{d}{dx}\left[x^2 + 2xy - 3y^2\right] = \frac{d}{dx}(-7)$$

$$2x + 2y + 2x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$$

$$(2x - 6y)\frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x - 6y}$$

OR

$$\boxed{\frac{dy}{dx} = \frac{x+y}{-x+3y}}$$

   (b) Find an equation of the line tangent to the graph the point $(x, y) = (1, 2)$.

$$\text{Slope} = m = \left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = \frac{1+2}{-1+3(2)} = \frac{3}{5}$$

$$y - 2 = \frac{3}{5}(x-1)$$

OR

$$y = \frac{3}{5}x + \frac{7}{5}$$

   (c) Find an equation of the line normal to the graph at the point $(x, y) = (1, 2)$.

NORMAL LINE IS PERP. TO TAN. LINE $\Rightarrow$

$$m = -\frac{5}{3}$$

$$y - 2 = -\frac{5}{3}(x-1)$$

OR

$$y = -\frac{5}{3}x + \frac{11}{3}$$

2. (2 points) Let $f(x) = x + \sqrt{x}$. Find $(f^{-1})'(2)$.

FROM OUR FORMULA...

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$$

$$\underbrace{f^{-1}(2) = y \Leftrightarrow f(y) = 2 \Leftrightarrow y + \sqrt{y} = 2 \Leftrightarrow y = 1}_{f^{-1}(2) = 1}$$

$$f'(x) = 1 + \frac{1}{2}x^{-1/2} \Rightarrow f'(1) = \frac{3}{2}$$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{3/2} = \boxed{\frac{2}{3}}$$

3. (3 points) Determine each derivative. Do not simplify, but show all your work.

(a) $\frac{d}{dx}\left[x \cot^{-1}(x^2)\right] = \cot^{-1}(x^2) + x \frac{d}{dx} \cot^{-1}(x^2)$

$$= \cot^{-1}(x^2) + x\left(\frac{-1}{1+x^4}\right)(2x)$$

$$= \cot^{-1}(x^2) - \frac{2x^2}{1+x^4}$$

(b) $\frac{d}{dt} e^{t^3 \ln t} = e^{t^3 \ln t} \frac{d}{dt}\left(t^3 \ln t\right)$

$$= e^{t^3 \ln t}\left(t^3\left(\frac{1}{t}\right) + 3t^2 \ln t\right)$$

$$= e^{t^3 \ln t}\left(t^2 + 3t^2 \ln t\right)$$

(c) $\frac{d}{dr}\frac{\ln(9r)}{2^r} = \frac{2^r\left(\frac{1}{9r}\right)(9) - \ln(9r)\, 2^r \ln 2}{(2^r)^2}$

$$= \frac{\frac{1}{r} - \ln(9r)\ln(2)}{2^r}$$

4. (2 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ when $y = x^{\sqrt{x}}$.

$$\ln y = \sqrt{x} \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2} x^{-1/2} \ln x + (\sqrt{x})\frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx} = x^{\sqrt{x}}\left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right)$$