Math 131 - Quiz 5

April 9, 2020

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than April 14.

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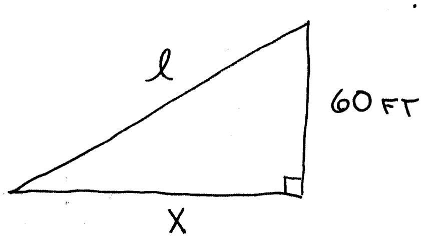
1. (3 points) A girl on flat ground flies a kite at a height of 60 ft. The wind carries the kite horizontally away from her at a rate of 5 ft/sec. How fast is the distance (diagonally) between the girl and the kite increasing when the kite is 150 ft away from her? Carefully work through the four steps of this related-rates problem.



$\frac{dx}{dt} = 5$ FT/s

FIND $\frac{d\ell}{dt}$ WHEN $\ell = 150$ FT

$x^2 + 60^2 = \ell^2$

$x^2 + 3600 = \ell^2$

$2x\frac{dx}{dt} = 2\ell\frac{d\ell}{dt}$

$\frac{d\ell}{dt} = \frac{x}{\ell}\frac{dx}{dt}$

WHEN $\ell = 150$

$x = \sqrt{150^2 - 60^2}$

$= \sqrt{18900} \approx 137.48$

$\frac{d\ell}{dt} = \frac{\sqrt{18900}}{150}(5)$

$\approx \boxed{4.58 \text{ FT/s}}$

2. (2 points) Let $f(x) = \frac{1}{x} + \sqrt[3]{x}$.

(a) Determine the linearization of f at $x = 8$. Write your answer in exact form (fractions, not decimals).

$f'(x) = -\frac{1}{x^2} + \frac{1}{3}x^{-2/3}$

$f'(8) = -\frac{1}{64} + \frac{1}{3}\left(\frac{1}{4}\right)$

$= \frac{1}{12} - \frac{1}{64} = \frac{13}{192}$

$L(x) = f(8) + f'(8)(x-8)$

$\boxed{L(x) = \frac{17}{8} + \frac{13}{192}(x-8)}$

(b) Use your linearization to approximate $f(8.1)$. Round to the 6th decimal place.

$f(8.1) \approx L(8.1) = \frac{17}{8} + \frac{13}{192}(8.1-8)$

$\approx \boxed{2.131771}$

3. (2 points) Determine the differential dy.

(a) $y = \sin^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}}(2x) \Rightarrow \boxed{dy = \frac{2x}{\sqrt{1-x^4}}\,dx}$$

(b) $y = x5^{-x^2}$

$$\frac{dy}{dx} = 5^{-x^2} + x\,5^{-x^2}(-2x)\ln 5$$

$$\boxed{dy = 5^{-x^2}(1-2x^2\ln 5)\,dx}$$

4. (3 points) Find the absolute extreme values of $g(x) = x^4 - 4x^3$ on the interval $[-1, 5]$.

$g'(x) = 4x^3 - 12x^2$

$g'(x) = 0 \Rightarrow 4x^2(x-3) = 0$

$x = 0,\ x = 3$

THE CRITICAL NUMBERS ARE $x=0$ & $x=3$.

INTERVAL ENDPOINTS ARE. $x=-1$ & $x=5$

$g(0) = 0$

$\boxed{g(3) = -27 \leftarrow \text{ABS. MIN.}}$

$g(-1) = 5$

$\boxed{g(5) = 125 \leftarrow \text{ABS. MAX.}}$