Math 131 - Quiz 6
April 23, 2020

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this quiz. This quiz is due no later than April 28.

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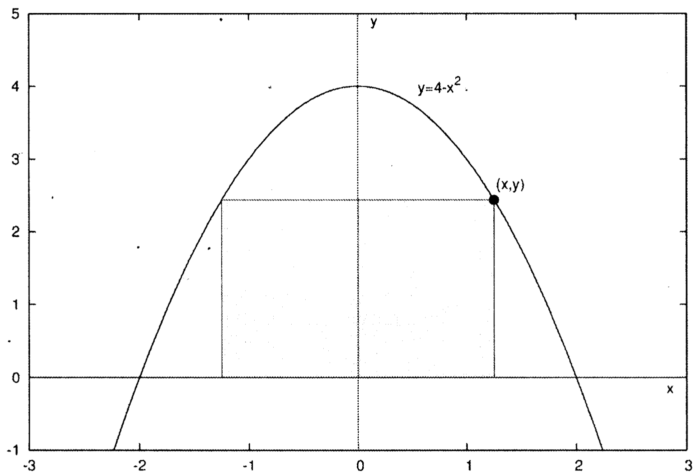
1. (4 points) A rectangle is bounded below by the x-axis and above by the graph of $y = 4 - x^2$ (see figure). Use calculus to find the coordinates of the point (x, y) that maximize the area of the rectangle.



THE AREA OF THE RECTANGLE IS $A = 2xy$, $0 \le x \le 2$.

WE ALSO KNOW THAT $y = 4 - x^2$. SO...

$$A = 2x(4 - x^2) = 8x - 2x^3, \quad 0 \le x \le 2$$

$$\frac{dA}{dx} = 8 - 6x^2 = 0 \Rightarrow x^2 = \frac{8}{6} \Rightarrow x = \sqrt{\frac{4}{3}}$$

CHECK CRIT PTS AND ENDPTS...

$A(0) = 0$ $\quad A\left(\sqrt{\frac{4}{3}}\right) = \frac{32}{3\sqrt{3}} \leftarrow$ ABS MAX

$A(2) = 0$

$$\boxed{x = \sqrt{\frac{4}{3}} \quad y = 4 - \frac{4}{3} = \frac{8}{3}}$$

2. (4 points) Compute each limit. Show all work!

(a) $\lim_{x\to 0} \dfrac{\arctan x}{\sin x}$ $\frac{0}{0}$ $\underbrace{=}_{\text{L' Hôpital's rule}} \lim_{x\to 0} \dfrac{\frac{1}{1+x^2}}{\cos x} = \dfrac{\frac{1}{1+0}}{1} = 1$

(b) $\lim_{x\to\infty} \dfrac{x^3}{e^{x^2}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \dfrac{3x^2}{2xe^{x^2}} = \lim_{x\to\infty} \dfrac{3x}{2e^{x^2}}$ $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \dfrac{3}{4xe^{x^2}} \overset{\frac{3}{\infty}}{=} 0$

L' Hôpital's rule two times!

3. (2 points) Try using L'Hôpital's rule to compute $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$. What happens? Can you determine the limit by using techniques we learned earlier?

First, the limit is 1. By the techniques of Section 4.6...

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \sqrt{\frac{x^2}{x^2+1}} \cdot \sqrt{\frac{\frac{1}{x^2}}{\frac{1}{x^2}}} = \lim_{x\to\infty} \sqrt{\frac{1}{1+\frac{1}{x^2}}} = \sqrt{\frac{1}{1+0}}$$

$$= 1$$

Now, L' Hôpital's Rule...

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{2}(x^2+1)^{-1/2}(2x)} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} \overset{\frac{\infty}{\infty}}{=}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{2}(x^2+1)^{-1/2}(2x)}{1} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

After L' Hôpital twice, we are back to where we started.