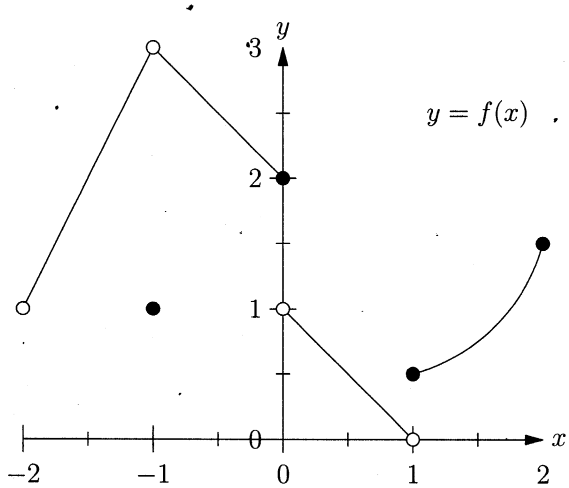
Math 131 - Test 1
February 20, 2020

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (10 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -1} f(x) = 3$

(b) $\lim_{x \to 0^-} f(x) = 2$

(c) $f(1) = \frac{1}{2}$

(d) $\lim_{x \to 0^+} f(x) = 1$

(e) $\lim_{x \to -2} f(x) =$ DNE. $\lim_{x \to -2^+} f(x) = 1$, BUT $\lim_{x \to -2^-} f(x)$ DNE SINCE f IS NOT DEFINED TO THE LEFT OF -2.

2. (9 points) Suppose that $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x)$ exists. Determine each limit.

(a) $\lim_{x\to 2} (x^2 f(x) + g(x)\sin \pi x)$

$$= \lim_{x\to 2} x^2 \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) \lim_{x\to 2} \sin \pi x$$

$$= (4)\ (3) + \left(\begin{smallmatrix}\text{SOME}\\ \#\end{smallmatrix}\right)(0) = \boxed{12}$$

(b) $\lim_{x\to 2} g(x)$ if $\lim_{x\to 2} \dfrac{1}{(g(x))^2} = 5$

$$\left[\lim_{x\to 2} g(x)\right]^2 = \frac{1}{5} \Rightarrow \lim_{x\to 2} g(x) = \boxed{\frac{1}{\sqrt{5}}} \text{ OR } \frac{-1}{\sqrt{5}}$$

(c) $\lim_{x\to 2} g(x)$ if $\lim_{x\to 2} \dfrac{f(x)}{g(x)}$ does not exist

ONLY WAY THIS CAN HAPPEN IS IF

$$\lim_{x\to 2} g(x) = \boxed{0}$$

3. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at four or more points.

$$\lim_{x\to 0^+} \frac{5^x - 1}{3x}$$

FROM THE RIGHT: $X > 0$

X	f(x)
0.01	0.5408
0.001	0.5369
0.0001	0.5365
0.00001	0.5365

$$\lim_{x\to 0^+} \frac{5^x - 1}{3x} \approx 0.5365$$

4. (6 points) Determine all points at which g is discontinuous. Carefully explain your reasoning.

$$g(x) = \begin{cases} x^2+2, & x \le 0 \\ 1+\cos x, & 0 < x < \pi \\ 1 + x\sin x, & x \ge \pi \end{cases}$$

BECAUSE THE PIECES ARE "NICE" CONT. FUNCTIONS,

THE ONLY POSSIBLE DISCONTS ARE AT $X = 0$ & $X = \pi$

$X = 0$

$$\lim_{x\to 0^-} g(x) = (0)^2 + 2 = 2$$

$$\lim_{x\to 0^+} g(x) = 1 + \cos(0) = 2$$

$$g(0) = (0)^2 + 2 = 2$$

CONTINUOUS AT $X = 2$

$X = \pi$

$$\lim_{x\to \pi^-} g(x) = 1 + \cos \pi = 0$$

$$\lim_{x\to \pi^+} g(x) = 1 + \pi \sin \pi = 1$$

LIMIT AT π DNE

$\Rightarrow$ $\boxed{\text{DISCONT. AT } X = \pi}$

5. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\displaystyle\lim_{x\to 0} \frac{(x-2)^2-4}{x}$ 0/0 MORE WORK

$$= \lim_{x\to 0} \frac{x^2-4x+4-4}{x} = \lim_{x\to 0} \frac{x^2-4x}{x} = \lim_{x\to 0} (x-4) = \boxed{-4}$$

(b) $\displaystyle\lim_{k\to 4} \frac{\sqrt{k}-2}{k-4} \cdot \frac{\sqrt{k}+2}{\sqrt{k}+2}$ 0/0 MORE WORK

$$= \lim_{k\to 4} \frac{k-4}{(k-4)(\sqrt{k}+2)} = \lim_{k\to 4} \frac{1}{\sqrt{k}+2} = \boxed{\frac{1}{4}}$$

(c) $\displaystyle\lim_{x\to 4^-} \left(\frac{x-4}{x^2-8x+16}\right)$ 0/0 MORE WORK

$$\lim_{x\to 4^-} \frac{x-4}{(x-4)^2} = \lim_{x\to 4^-} \frac{1}{x-4}$$

1/0 FORM. $\pm\infty$?

TO THE LEFT OF 4
X − 4 IS NEG $\Rightarrow$ LIMIT IS $\boxed{-\infty}$

(d) $\displaystyle\lim_{z\to 6} \frac{(z-4)^2+2(z+1)}{z+3} = \frac{2^2+14}{9} = \frac{18}{9} = \boxed{2}$

6. (4 points) Given that $-x^2 \le x^2 \cos\frac{1}{x} \le x^2$ when $x \neq 0$, compute $\lim_{x\to 0} x^2 \cos\frac{1}{x}$. Explain. (Can you state the name of the theorem you used?)

$\lim_{x\to 0}(-x^2) = 0$ AND $\lim_{x\to 0} x^2 = 0$

$x^2 \cos\frac{1}{x}$ IS BETWEEN $-x^2$ & x^2.

$\therefore \lim_{x\to 0} x^2 \cos\frac{1}{x} = 0$

SQUEEZE THEOREM

7. (5 points) Use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to compute $\lim_{x\to 0}\left(\frac{3\tan 2x}{x}\right)$.

$$\lim_{x\to 0} \frac{3 \sin 2x}{x \cos 2x} = \lim_{x\to 0} \frac{6 \sin 2x}{2x \cos 2x}$$

$$= \lim_{x\to 0} (6)\left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{\cos 2x}\right)$$

$$= (6)(1)(1) = \boxed{6}$$

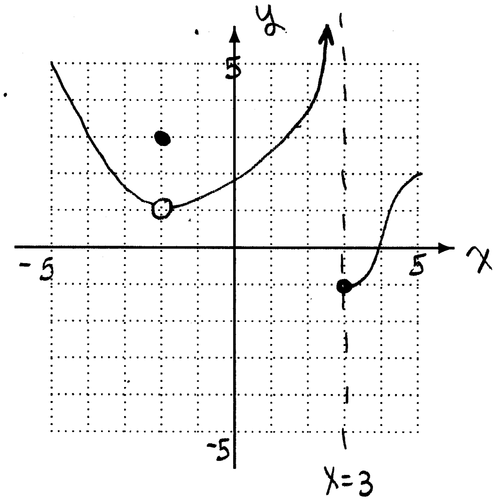
8. (5 points) Each row of the table below gives some information about a function f. Fill in each blank entry with an appropriate word or number. In some cases there may be more than one correct answer.

Continuous at $x = 2$	$f(2)$	$\lim_{x\to 2^-} f(x)$	$\lim_{x\to 2^+} f(x)$
Yes	5	5	5
No	7	Any # $\neq$ 7	7
No	Any # $\neq$ -1	-1	-1
Yes	2	2	2
Yes	1	1	1

9. (6 points) Sketch the graph of a function f such that

- f is defined for all real numbers between -5 and 5,
- $f(-2) = 3$,
- f has a removable discontinuity at $x = -2$,
- $\lim_{x \to 3^-} f(x) = \infty$, and
- $\lim_{x \to 3^+} f(x) = -1$.

LOTS OF POSSIBLE ANSWERS.



10. (6 points) Consider the rational function $R(x) = \dfrac{2x+4}{x^2+3x+2}$. Find all points at which R is discontinuous, and state whether each discontinuity is removable or non-removable.

$$R(x) = \frac{2(x+2)}{(x+2)(x+1)}$$

R IS CONT. EVERYWHERE IT IS DEFINED

$\Rightarrow$ R IS DISCONTINUOUS ONLY AT $X=-2$ & $X=-1$

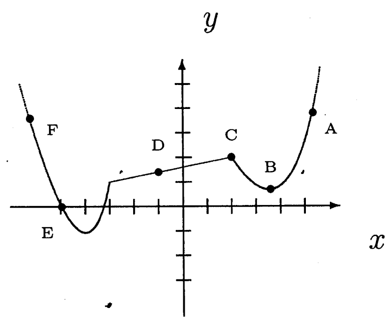
$$\lim_{X \to -2} R(x) = \lim_{X \to -2} \frac{2}{X+1} = -2$$

DISCONT AT $X=-2$ IS REMOVABLE.

$\lim_{X \to -1} R(x)$ HAS "FORM" $\frac{\text{NONZERO}}{0}$

DISCONT. AT $X=-1$ IS INFINITE (NON-REMOVABLE).

11. (6 points) Consider the function f whose graph is shown below.



Referring to the labeled points, find a point at which

(a) $f'(x) = 0$

B

(b) $0 < f'(x) < 1$ D

(c) $f'(x) > 1$ A

(d) $f(x) = 0$ E

(e) $f'(x) < 0$ E or F

(f) $f'(x)$ is not defined C

Formal Definition of Limit: The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - c| < \delta$ implies that $|f(x) - L| < \epsilon$.

12. (5 points) Compute $\lim_{x \to -2} (2x - 5)$ and then, referring to the formal definition of limit, find a δ that corresponds to an arbitrary positive ϵ.

$$\lim_{x \to -2} (2x-5) = -9$$

LET ϵ BE AN ARBITRARY POSITIVE #.

$$|(2x-5)-(-9)| < \epsilon$$

$$\Uparrow$$

$$|2x+4| < \epsilon$$

$$2|x+2| < \epsilon$$

$$\Uparrow$$

$$|x+2| < \frac{\epsilon}{2}$$

$$\delta = \frac{\epsilon}{2}$$

13. (8 points) Let $f(x) = x^2 - 2x$. Use the limit definition of derivative to determine $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 - 2(x+h)] - [x^2 - 2x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- 2x} - 2h \cancel{- x^2} \cancel{+ 2x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h - 2) = 2x - 2$$

$$\boxed{f'(x) = 2x - 2}$$