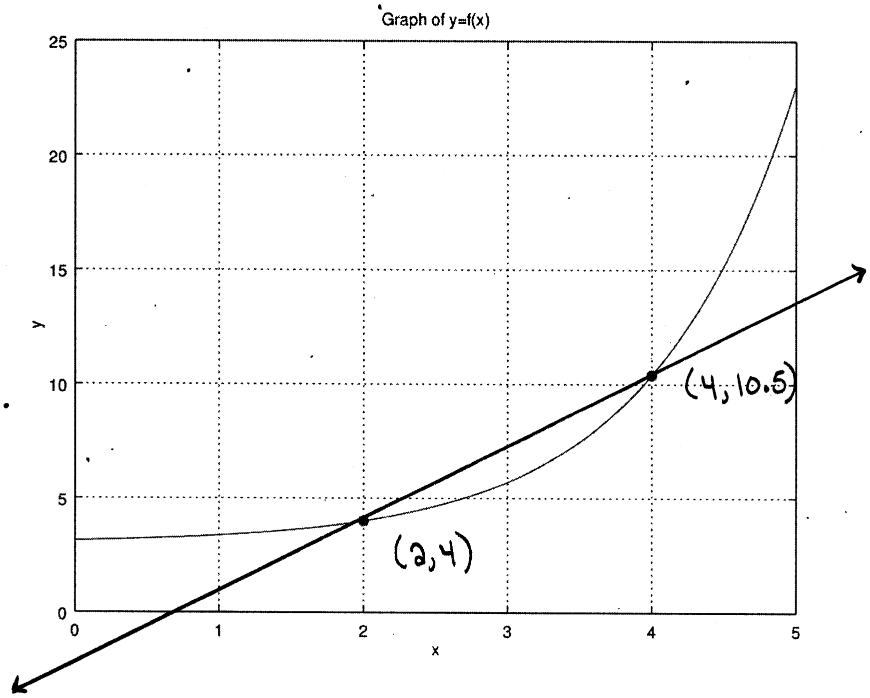
Math 131 - Test 2
March 12, 2020

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Use differentiation rules for all derivatives, and do not simplify.

1. (6 points) The graph of $y = f(x)$ is shown below.



(a) Sketch the secant line through the indicated points at $x = 2$ and $x = 4$. Let m be the slope of the secant line through those points. Estimate the value of m.

$$m \approx \frac{10.5 - 4}{4 - 2} = \frac{6.5}{2} = \boxed{3.25}$$

(b) Which number is greatest: m, $f'(2)$, or $f'(4)$?. Explain your reasoning.

$\boxed{f'(4).}$ THE TANGENT LINE AT $X=4$ IS STEEPER THAN THE SECANT LINE AND THE TAN. LINE AT $X=2$.

(c) Which number is least: m, $f'(2)$, or $f'(4)$?. Explain your reasoning.

$\boxed{f'(2).}$ THE TANGENT LINE AT $X=2$ IS FLATTER THAN THE SECANT LINE AND THE TAN. LINE AT $X=4$.

2. (4 points) Which one of the following best describes the line tangent to the graph of $f(x) = 5x^{1/3} - 2$ at the point $(0, -2)$? (Briefly explain, or show work, to receive full credit.)

(a) The tangent line is horizontal.

(b) The tangent line is vertical.

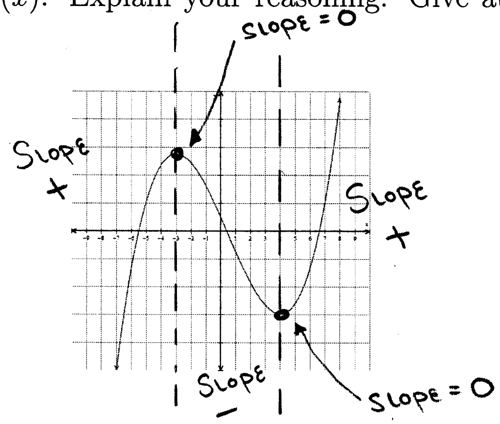
(c) A unique tangent line does not exist.

(d) The tangent line cannot be determined from the given information.

$$f'(x) = \frac{5}{3} x^{-2/3} = \frac{5}{3x^{2/3}}$$

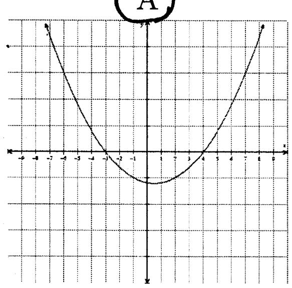
$\therefore$ As $x \to 0$, $f'(x) \to \infty$

3. (5 points) The graph of $g(x)$ is shown below. Choose the lettered graph that best represents the graph of $g'(x)$. Explain your reasoning. Give at least two reasons to support your answer.

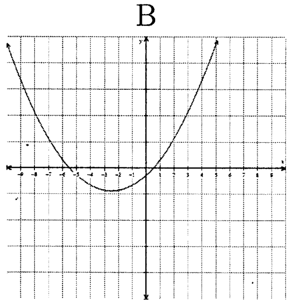


$g'(x)$

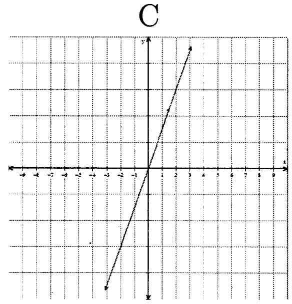
A



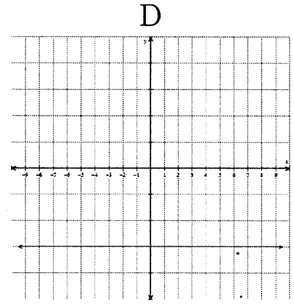
B



C



D



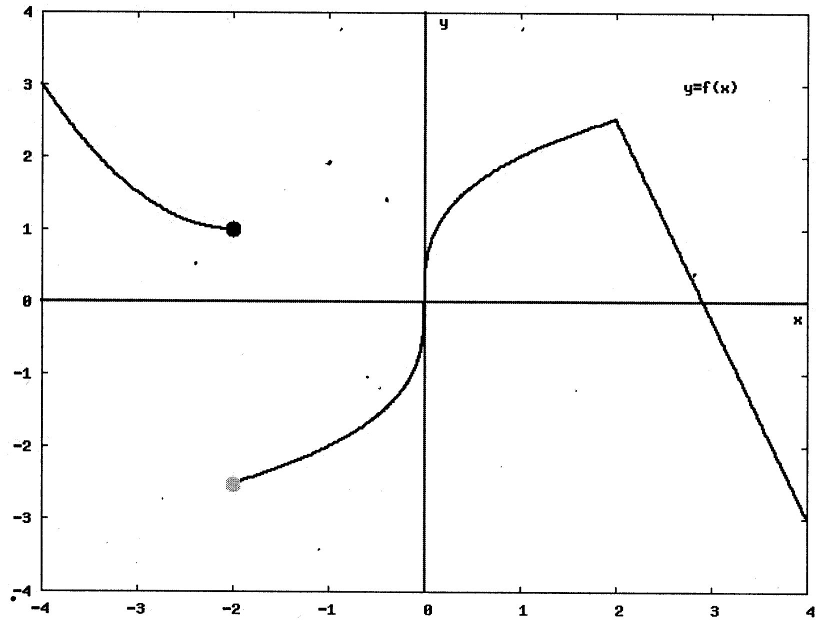
Reasons:

① Graph of g has horizontal tangent lines at $x = -3$ & $x = 4$. $g'(x) = 0$ at those points.

② Tangent lines for g have positive slope for $x < -3$ and $x > 4$. $g'(x)$ is positive at those pts.

③ Similar for neg. slopes between $x = -3$ & $x = 4$

4. (6 points) The graph of $y = f(x)$ is shown below. Give the x-coordinates of three points at which $f'(x)$ does not exist. For each point, very briefly say why f' does not exist.



$X = -2$; f IS DISCONTINUOUS; TANGENT LINE CANNOT EXIST

$X = 0$; TANGENT LINE APPEARS TO BE VERTICAL

$X = 2$; SHARP POINT; DIFFERENT TANGENT LINES FROM OPPOSITE SIDES.

5. (5 points) Use the quotient rule to derive the formula for the derivative of $y = \cot x$.

$$\frac{d}{dx} \cot x = \frac{d}{dx} \frac{\cos x}{\sin x} = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x$$

6. Suppose you launch an object straight upward with a velocity of 64 ft/sec from over the edge of the top of an 80-ft building. Use the position function

$$s(t) = -16t^2 + v_0 t + s_0,$$

where s represents height (in feet) at time t (in seconds), to solve the following problems.

(a) (2 points) Determine the function that gives the object's height at time t.

$$S(t) = -16t^2 + 64t + 80$$

(b) (2 points) Determine the average rate of change the object's height over the interval from $t = 0$ to $t = 2$. (Include units with your answer.)

$$\frac{\Delta S}{\Delta t} = \frac{S(2) - S(0)}{2 - 0} = \frac{144 - 80}{2} = 32 \text{ FT/SEC}$$

(c) (2 points) Determine the function that gives the object's velocity at time t.

$$V(t) = S'(t) = -32t + 64$$

(d) (2 points) Determine the object's velocity after 4 seconds. (Include units with your answer.)

$$V(4) = -32(4) + 64 = -64 \text{ FT/SEC}$$

(e) (2 points) What is the acceleration of the object? (Include units with your answer.)

$$V'(t) = a(t) = S''(t) = -32 \text{ FT/SEC}^2$$

(f) (4 points) Determine the object's maximum height. (Include units with your answer.)

$$V(t) = 0 \Rightarrow -32t + 64 = 0$$
$$t = 2$$

$$S(2) = 144 \text{ FT}$$

(g) (3 points) When does the object hit the ground?

$$S(t) = 0 \Rightarrow -16t^2 + 64t + 80 = 0$$
$$-16(t^2 - 4t - 5) = 0$$
$$-16(t-5)(t+1) = 0$$

$$t = 5 \text{ SEC}$$

7. (6 points) Use the derivative to determine each point (both coordinates) on the graph of $y = x^4 - 2x^2 + 3$ at which the tangent line is horizontal.

$\frac{dy}{dx} = 4x^3 - 4x$

HORIZ. TAN. LINE

$\Rightarrow \frac{dy}{dx} = 0$

$4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$4x(x-1)(x+1) = 0$

$x = 0, \quad x = 1, \quad x = -1$

$(0,3), \quad (1,2) \quad (-1,2)$

8. (6 points) Find an equation of the line tangent to the graph of $y = \frac{x^3 - 3x^2 + 4}{x^2}$ at the point where $x = 1$.

SLOPE... $y = x - 3 + 4x^{-2}$

$\frac{dy}{dx} = 1 - 8x^{-3}$

$m = \left.\frac{dy}{dx}\right|_{x=1} = 1 - 8 = -7$

POINT... $x = 1, \; y = 2$

$y - 2 = -7(x-1)$

OR

$y = -7x + 9$

9. (7 points) Let $h(x) = \sqrt{x^4 + 9}$. Identify two functions, f and g, so that $h(x) = f(g(x))$. Then use the chain rule to determine $h'(x)$.

$g(x) = x^4 + 9$

$f(x) = \sqrt{x} = x^{1/2}$

$h'(x) = f'(g(x))\,g'(x)$

$h'(x) = \frac{1}{2}(x^4+9)^{-1/2}(4x^3)$

$h'(x) = \frac{2x^3}{\sqrt{x^4+9}}$

10. (6 points) Let $f(x) = 3x \sin x$. Find $f''(x)$.

$f'(x) = 3 \sin x + 3x \cos x$

$f''(x) = 3 \cos x + 3 \cos x + 3x(-\sin x)$

$\boxed{f''(x) = 6 \cos x - 3x \sin x}$

11. (12 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	-2	-1	2
$f(x)$	1	3	-2
$f'(x)$	2	-1	-1
$g(x)$	2	0	-2
$g'(x)$	-3	-2	1

(a) If $h(x) = 4f(x) - 2g(x) + 5$, compute $h'(-1)$.

$h'(x) = 4f'(x) - 2g'(x)$

$h'(-1) = 4(-1) - 2(-2) = -4 + 4 = \boxed{0}$

(b) If $h(x) = f(x) \cdot g(x)$, compute $h'(-1)$.

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(-1) = f'(-1)g(-1) + f(-1)g'(-1)$

$= (-1)(0) + (3)(-2) = \boxed{-6}$

(c) If $h(x) = \dfrac{3f(x)}{g(x)}$, compute $h'(2)$.

$$h'(x) = \frac{3f'(x)g(x) - 3f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{3f'(2)g(2) - 3f(2)g'(2)}{[g(2)]^2} = \frac{(3)(-1)(-2) - (3)(-2)(1)}{(-2)^2}$$

$$= \frac{12}{4} = \boxed{3}$$

12. (20 points) Differentiate. Do not simplify.

(a) $\frac{d}{dr}\left(7r^5 + 4r - 8\sqrt{r} + \frac{17}{r^2}\right)$

$$\frac{d}{dr}\left(7r^5 + 4r - 8r^{1/2} + 17r^{-2}\right) = \boxed{35r^4 + 4 - 4r^{-1/2} - 34r^{-3}}$$

(b) $\frac{d}{dx}\left[(x^3 - x)\sec x\right]$

PRODUCT RULE...

$$\boxed{(3x^2 - 1)\sec x + (x^3 - x)\sec x \tan x}$$

(c) $\frac{d}{dx}\cos(x^2+1)$

CHAIN RULE...

$$-\sin(x^2+1)(2x) = \boxed{-2x\sin(x^2+1)}$$

(d) $\frac{d}{dt}(2t+1)^5(3t-2)^7$.

PRODUCT RULE & CHAIN RULE...

$$\left[\frac{d}{dt}(2t+1)^5\right](3t-2)^7 + (2t+1)^5\left[\frac{d}{dt}(3t-2)^7\right]$$

$$\boxed{\left[5(2t+1)^4(2)\right](3t-2)^7 + (2t+1)^5\left[7(3t-2)^6(3)\right]}$$