Math 131 - Test 3

April 16, 2020

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this test. Submit your test in Blackboard no later than Monday, April 20, at 9pm.

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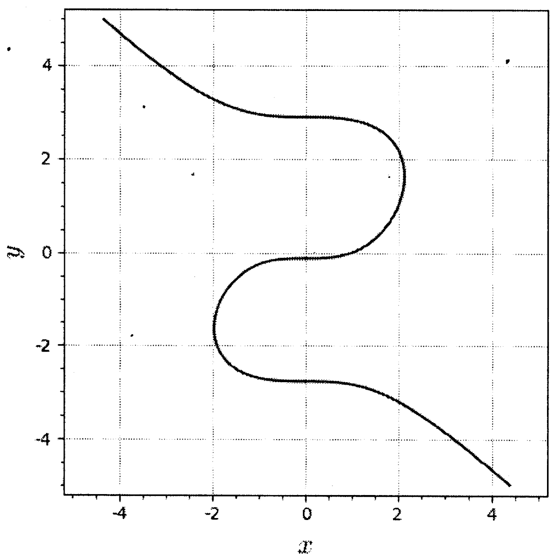
1. (12 points) The graph of the equation $x^3 + y^3 = 8y + 1$ is shown below.



(a) Use implicit differentiation to find a formula for dy/dx.

$$\frac{d}{dx}\left(x^3 + y^3\right) = \frac{d}{dx}\left(8y+1\right)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 8\frac{dy}{dx}$$

$$3x^2 = \left(8 - 3y^2\right)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{8-3y^2}}$$

(b) Use dy/dx to compute the slope of the graph at the point $(2, 1)$. Then determine an equation of the tangent line at $(2, 1)$.

$$m = \left.\frac{dy}{dx}\right|_{(2,1)} = \frac{3(4)}{8-3(1)} = \frac{12}{5}$$

$$y - 1 = \frac{12}{5}(x-2)$$

OR

$$y = \frac{12}{5}x - \frac{19}{5}$$

2. (5 points) Let $f(x) = x^3 + x + 1$. It is easy to see that $f^{-1}(3) = 1$ (because $f(1) = 3$). Determine the value of $(f^{-1})'(3)$.

$$f'(x) = 3x^2 + 1$$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \boxed{\frac{1}{4}}$$

3. (5 points) Find the slope of the line tangent to the graph of $y = \sin^{-1}(x/2)$ at the point $(1, \pi/6)$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \left(\frac{1}{2}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{\sqrt{\frac{3}{4}}} \cdot \left(\frac{1}{2}\right)$$

$$= \boxed{\frac{1}{\sqrt{3}}}$$

4. (5 points) Let $f(x) = e^{5^x}$. Compute $f'(1)$. Round you final answer to two decimal places.

$$f'(x) = e^{5^x} \cdot 5^x \ln 5$$

$$f'(1) = e^5 \cdot 5 \cdot \ln 5 \approx 1194.31$$

5. (5 points) Let $g(t) = 2\ln(t^2+1)$. Determine $g'(t)$.

$$g'(t) = 2\left(\frac{1}{t^2+1}\right)(2t)$$

$$= \boxed{\frac{4t}{t^2+1}}$$

6. (5 points) Suppose that $y = \dfrac{x^3}{\sqrt{x+1}}$. Carry out the first two steps in the process of logarithmic differentiation, and then stop.

1. $$\ln y = \ln\left(\frac{x^3}{\sqrt{x+1}}\right)$$

2. $$\ln y = \ln x^3 - \ln\sqrt{x+1}$$

$$\ln y = 3\ln x - \frac{1}{2}\ln(x+1)$$

7. (5 points) A particle is moving along the graph of $y = \sqrt{x}$ in such a way that $\dfrac{dx}{dt} = 5$. Find $\dfrac{dy}{dt}$ when $x = 9$.

$$y = \sqrt{x}$$

$$\frac{d}{dt}y = \frac{d}{dx}(x^{1/2})$$

$$\frac{dy}{dt} = \frac{1}{2}x^{-1/2}\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=9} = \frac{1}{2}(9)^{-1/2}(5)$$

$$= \boxed{\frac{5}{6}}$$

8. (10 points) Some values of $f(x)$ and $f'(x)$ near $x = 1$ are given in the table below.

x	0.50	0.75	1.00	1.25	1.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

(a) Determine the linearization of f at $x = 1$.

$$L(x) = f(1) + f'(1)(x-1)$$

$$\boxed{L(x) = 8 + 5(x-1)}$$

(b) Use the linearization you found above to approximate $f(0.75)$.

$$f(0.75) \approx L(0.75) = 8 + 5(0.75-1)$$

$$= 8 + 5(-0.25)$$

$$= \boxed{6.75}$$

9. (5 points) Let $y = (1 + \sin x)^4$. Determine the differential dy.

$$\frac{dy}{dx} = 4(1+\sin x)^3 (\cos x)$$

$$\boxed{dy = 4\cos x\,(1+\sin x)^3\,dx}$$

10. (10 points) Find the absolute maximum and minimum values of $g(x) = x^3 - 3x^2 + 1$ on $[-1/2, 4]$.

$g'(x) = 3x^2 - 6x$

CRIT. NUMBERS ...

$g'(x) = 0 \Rightarrow 3x(x-2) = 0.$

$x = 0,\ x = 2$

$g'(x)$ DNE NOWHERE.

ONLY CRIT. NUMBERS ARE

$x = 0, 2$

CHECK CRIT. NUMBERS AND ENDPOINTS ...

$g(0) = 1$

$\boxed{g(2) = -3 \leftarrow \text{ABS MIN}}$

$g(-\frac{1}{2}) = \frac{1}{8}$

$\boxed{g(4) = 17 \leftarrow \text{ABS MAX}}$

11. (12 points) Let $r(x) = x^3 + \sin(10x)$.

(a) By using the 1st derivative, determine whether r is increasing or decreasing at the point where $x = 0.65$.

$r'(x) = 3x^2 + 10\cos 10x$

$r'(0.65) \approx 11.03337 > 0 \Rightarrow$ $\boxed{r \text{ IS INCREASING.}}$

(b) By using the 2nd derivative, determine whether the graph of r is concave up or concave down at the point where $x = 0.65$.

$r''(x) = 6x - 100\sin 10x$

$r''(0.65) \approx -17.61199 < 0 \Rightarrow$ $\boxed{\text{GRAPH IS CONCAVE DOWN}}$

12. (5 points) What is an inflection point?

IT IS A POINT ON A GRAPH WHERE THE CONCAVITY CHANGES.

13. (10 points) Let $f(x) = x^4 + 4x^3 - 36x^2$.

(a) Find the critical numbers of f.

$f'(x) = 4x^3 + 12x^2 - 72x$

$f'(x)$ DNE NOWHERE.

$f'(x) = 0 \Rightarrow 4x(x^2 + 3x - 18) = 0$

$4x(x+6)(x-3) = 0$

CRITICAL NUMBERS ARE

$x = 0, \ x = -6, \ x = 3$

(b) The graph of f has two inflection points. Find their x-coordinates.

$f''(x) = 12x^2 + 24x - 72$

$= 12(x^2 + 2x - 6)$

INFLECTION POINTS WILL OCCUR WHERE $f''(x) = 0$

OR

$x^2 + 2x - 6 = 0$

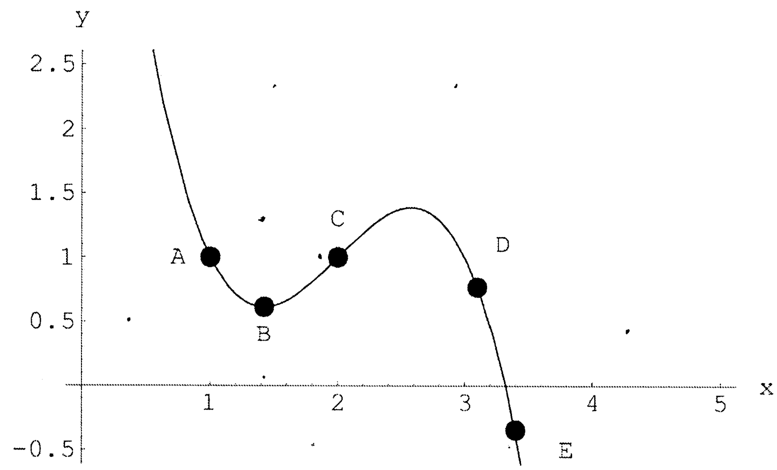
$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(-6)}}{2}$

$= \dfrac{-2 \pm \sqrt{28}}{2} = -1 \pm \sqrt{7}$

$x = -1 + \sqrt{7}$, $x = -1 - \sqrt{7}$

≈ 1.646 ≈ -3.646

14. (6 points) The graph of f is shown below. For each part of this problem, find a labeled point that satisfies the given condition.



(a) $f''(x) = 0$ C (INFLECTION POINT)

(b) $f'(x) = 0$ B (HORIZONTAL TANGENT LINE)

(c) $f''(x) < 0$ D, E (CONCAVE DOWN)

(d) $f(x) < 0$ E (y-VALUE IS NEG.)

(e) $f'(x) > 0$ C (INCREASING)

(f) $f''(x) > 0$ A, B (CONCAVE UP)