Math 131 - Final Exam

May 12, 2020

Name key

Score ____

Show all work to receive full credit. This test is due no later than May 16 at 8 am.

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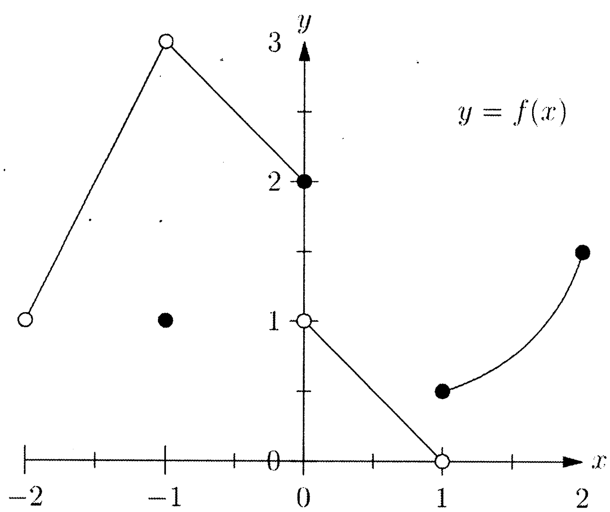
1. The graph of the function f is shown below. Use the graph to compute the following sum. If the sum does not exist, say why.

$$\lim_{x\to -2^+} f(x) + \lim_{x\to -1^-} f(x) + \lim_{x\to 1^+} f(x)$$

$1 + 3 + 0.5 = 4.5$



$\boxed{4.5}$

2. Find a point at which g is discontinuous. Using the definition of continuity at a point, say why g is discontinuous at your point.

$$g(x) = \begin{cases} x^2 + x, & x \le 1 \\ 2 + \sin(\pi x), & 1 < x \le 3 \\ 2^x - x - 1, & x > 3 \end{cases}$$

$g(3) = 2 + \sin 3\pi = 2$

$\lim_{x\to 3^+} g(x) = 2^3 - 3 - 1 = 4$

$\lim_{x\to 3^-} g(x) = 2 + \sin 3\pi = 2$

DISCONT AT $X = 3$ (JUMP) $\quad g(3) \neq \lim_{x\to 3} g(x)$

3. Use algebraic techniques (not L'Hôpital's rule) to evaluate the following limit.

$$\lim_{x\to 1} \frac{(x+1)^2 - 2x - 2}{x(2x-1) - 1} \quad \frac{0}{0}$$

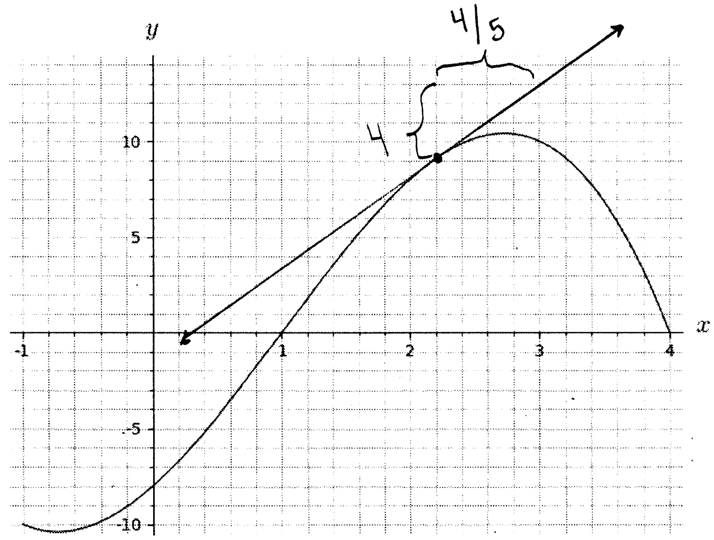
$$= \lim_{x\to 1} \frac{x^2 + 2x + 1 - 2x - 2}{2x^2 - x - 1}$$

$$= \lim_{x\to 1} \frac{x^2 - 1}{2x^2 - x - 1} \quad \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(2x+1)} = \frac{2}{3}$$

$$\boxed{\frac{2}{3}}$$

4. The graph of f is shown below. Sketch the tangent line at $x = 2$ and use it to estimate $f'(2)$.



$$f'(2) = m \approx \frac{4}{4/5}$$

$$f'(2) \approx 5$$

$$\boxed{f'(2) \approx 5}$$

5. Use the information in the table below to compute $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ at $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-7	0	4	2
1	-8	-1	7	5
2	-7	4	DNE	DNE
3	2	15	1	5

$$\frac{f(3)g'(3) - g(3)f'(3)}{[f(3)]^2} = \frac{(2)(5) - (1)(15)}{(2)^2}$$

$$= \frac{10-15}{4}$$

$$= -\frac{5}{4}$$

$$\boxed{-\frac{5}{4}}$$

6. An object is launched straight upward from the ground with an initial velocity of 96 feet per second. What is the height of the object at the moment its velocity is 16 feet per second? (Use $g = 32$ ft/sec^2.)

$s(t) = -16t^2 + 96t$ $\quad$ $v(t) = -32t + 96 = 16$

$\Rightarrow t = 2.5$

$s(2.5) = -16(2.5)^2 + 96(2.5)$

$= 140$

$$\boxed{140 \text{ FT}}$$

7. Let $f(x) = x\ln(x^2 + x)$. Determine $f'(2)$. Write your final answer in decimal form.

$$f'(x) = \ln(x^2+x) + x\left(\frac{1}{x^2+x}\right)(2x+1)$$

$$= \ln(x^2+x) + \frac{2x^2+x}{x^2+x}$$

$f'(2) = \ln 6 + \frac{10}{6}$

≈ 3.458426

$$\boxed{3.45843}$$

8. Given that $x \cos y = 1$, use implicit differentiation to find dy/dx at the point $(2, \pi/3)$. Write your answer in exact form.

$$\frac{d}{dx}(x \cos y) = 0 \Rightarrow \cos y + x(-\sin y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\cos y}{x \sin y} = \frac{\cot y}{x}$$

$$\boxed{\frac{\cot \frac{\pi}{3}}{2} = \frac{\sqrt{3}}{6}}$$

9. Suppose that the infected region of an injury is circular, and its radius is growing at the rate of 1.2 mm/hr. Find the rate of change of the area of the infected region when the radius is 3.4 mm.

$A = \pi r^2$, $\frac{dr}{dt} = 1.2$, FIND $\frac{dA}{dt}$ WHEN $r = 3.4$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ WHEN $r = 3.4$ ... $\frac{dA}{dt} = 2\pi(3.4)(1.2)$

$= 8.16\pi$

$$\boxed{8.16\pi \frac{mm^2}{HR} \approx 25.63539 \frac{mm^2}{HR}}$$

10. Look back at problem 5. Use the information in the table to find the linearization of f at $x = 3$. Then use your linearization to approximate $f(3.1)$.

$L(x) = f(3) + f'(3)(x-3)$

$L(x) = 2 + 15(x-3)$, $f(3.1) \approx L(3.1) = 3.5$

$$\boxed{L(x) = 2 + 15(x-3), \quad f(3.1) \approx 3.5}$$

11. Use calculus to find the absolute minimum value of $g(x) = e^{4x-x^2}$ on the interval $[1, 4]$.

$g'(x) = e^{4x-x^2}(4-2x)$

$g'(x) = 0 \Rightarrow x = 2$

$g(2) = e^4 \approx 54.6$

$g(1) = e^3 \approx 20.1$

$g(4) = e^0 = 1 \leftarrow$ MIN VALUE

$\boxed{\text{MIN VALUE IS } g(4) = 1}$

12. Determine the open interval(s) on which the graph of $h(x) = x^3 - 6x^2 + 12x$ is concave up.

$h'(x) = 3x^2 - 12x + 12$

$h''(x) = 6x - 12$

$h''(x) = 0 \Rightarrow x = 2$

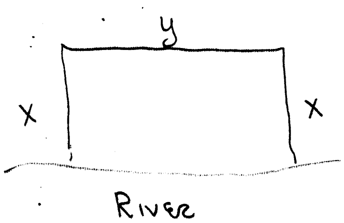
$h''(x)$: Neg. | 2 | Pos.

TEST $x = 0$ TEST $x = 3$

CU ON $(2, \infty)$

$\boxed{(2, \infty)}$

13. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must have an area of 88,200 square meters. No fencing will be used along the river. What dimensions will require the least amount of fencing?



MINIMIZE $F = 2x + y$ SUBJECT TO $xy = 88200$

$y = \dfrac{88200}{x}$

$F = 2x + \dfrac{88200}{x}$

$F' = 2 - \dfrac{88200}{x^2} = 0 \Rightarrow x = \sqrt{44100} = 210$

$\boxed{x = 210 \text{ FT}, \quad y = 420 \text{ FT}}$

$F'' = \dfrac{2(88200)}{x^3} > 0$ WHEN $x = 210$

$\Rightarrow 210$ GIVES A MIN.

$\dfrac{88200}{210} = 420$

14. Use L'Hôpital's rule to find the limit: $\lim_{x\to 0} \frac{6\tan 2x}{\sin 8x}$ $\quad \frac{0}{0}$

$$\lim_{x\to 0} \frac{12\sec^2 2x}{8\cos 8x} = \frac{12}{8} = \frac{3}{2}$$

$$\boxed{\frac{3}{2}}$$

15. Suppose B is a function that satisfies $\frac{d}{dx}\left(\frac{\sin x^3}{e^{3x}}\right) = B(x)$. Evaluate the following indefinite integral.

$$\int (x + 2\cos x - 7B(x))\,dx$$

$$\int x\,dx + 2\int \cos x\,dx - 7\int B(x)\,dx$$

$$\frac{1}{2}x^2 + 2\sin x - 7\left(\frac{\sin x^3}{e^{3x}}\right) + C$$

$$\boxed{\frac{1}{2}x^2 + 2\sin x - 7\left(\frac{\sin x^3}{e^{3x}}\right) + C}$$

16. Use 4 subintervals of equal length and right endpoints of the subintervals to compute a Riemann sum for $f(x) = x^2$ on $[0, 2]$. Write your final answer in decimal form.

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$, PARTITION IS $0 < 0.5 < 1 < 1.5 < 2$

$c_1 = 0.5$, $c_2 = 1$, $c_3 = 1.5$, $c_4 = 2$

$$\sum_{n=1}^{4} f(c_n)\Delta x = \left[(0.5)^2 + (1)^2 + (1.5)^2 + (2)^2\right]\left(\frac{1}{2}\right)$$

$= 3.75$

$$\boxed{3.75}$$

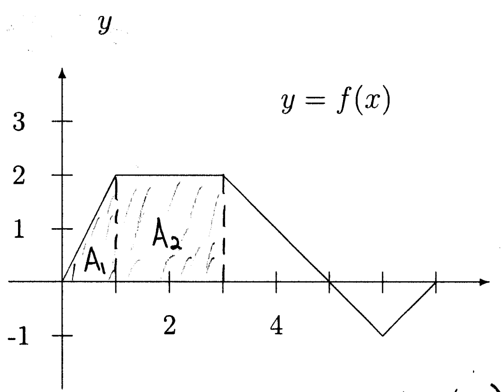
17. Suppose you know that $\int_1^3 3x^2\,dx = R$ and $\int_1^3 2x\,dx = S$. What is the value of $\int_3^1 (x^2 - x)\,dx$ in terms of R and S?

$$\int_1^3 x^2\,dx = \frac{R}{3} \qquad \int_1^3 x\,dx = \frac{S}{2}$$

$$\int_3^1 (x^2-x)\,dx = -\int_1^3 (x^2-x)\,dx = -\left[\frac{R}{3} - \frac{S}{2}\right]$$

$$\boxed{-\frac{R}{3} + \frac{S}{2}}$$

18. Given the graph of the function f, use area to compute $\int_0^3 f(x)\,dx$.

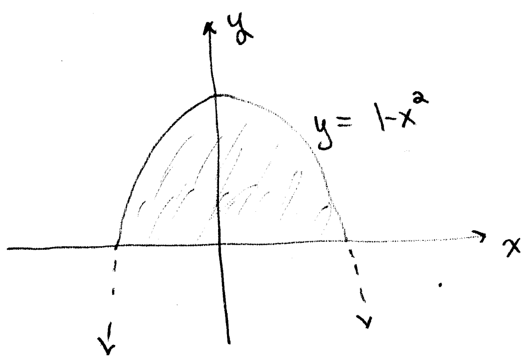


$$A_1 = \frac{1}{2}(1)(2) = 1$$

$$A_2 = (2)(2) = 4$$

$$\boxed{\int_0^3 f(x)\,dx = 1+4 = 5}$$

19. Use a definite integral to find the area of the bounded region between the graph of $y = 1 - x^2$ and the x-axis. Write your answer in exact form.



$$\text{AREA} = \int_{-1}^{1} (1-x^2)\,dx$$

$$= x - \frac{1}{3}x^3 \Big|_{-1}^{1}$$

$$= \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)$$

$$= \frac{4}{3}$$

$1-x^2=0 \Rightarrow x = \pm 1$

$$\boxed{\frac{4}{3}}$$

20. In order to evaluate the following integral, an appropriate u-substitution should be made. Carry out the substitution and write the new integral. Do not evaluate the new integral.

$$\int 7x^2 \sec(4x^3+1)\,dx$$

$u = 4x^3 + 1$

$du = 12x^2\,dx$

$\frac{1}{12}\,du = x^2\,dx$

$$\int \frac{7}{12} \sec u \, du$$

$$\boxed{\frac{7}{12}\int \sec u \, du}$$