Math 131 - Homework 2

February 17, 2021

Name key

Score ______

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due on February 24.

1. (2 points) Use the definition of the derivative to find $f'(3)$ if $f(x) = \frac{3}{x^2}$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{3x^2 - 3(x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0} \frac{3x^2 - 3x^2 - 6xh - 3h^2}{hx^2(x+h)^2}.$$

$$= \lim_{h\to 0} \frac{-6xh - 3h^2}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-3h(2x+h)}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-3(2x+h)}{x^2(x+h)^2}$$

$$= \frac{-6x}{x^4} = \frac{-6}{x^3} \qquad f'(x) = -\frac{6}{x^3} \Rightarrow \boxed{f'(3) = \frac{-6}{27} = -\frac{2}{9}}$$

2. (2 points) In the online version of the textbook, do problem 39 in section 3.1.

a) $f'(1) = \text{slope at } x = 1 = \frac{6-1}{4-0} = \boxed{\frac{5}{4}}$

b) $f'(6) = \text{slope at } x = 6 = \frac{8-6}{8-4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

Turn over.

3. (2 points) Use the definition of the derivative to find $f'(x)$ if $f(x) = 5x - x^2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left[5(x+h)-(x+h)^2\right] - \left[5x-x^2\right]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x}+5h-x^2-2xh-h^2-\cancel{5x}+x^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(5-2x-h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (5-2x-h) = 5-2x \Rightarrow \boxed{f'(x) = 5-2x}$$

4. (2 points) In the online version of the textbook, do problem 80 in section 3.2.

a) $f'(-0.5) = \frac{3-0}{0-(-3)} = \boxed{1}$

b) $f'(0)$ $\boxed{\text{DNE}}$ (SHARP POINT)

c) $f'(1) = \frac{1-3}{2-0} = \boxed{-1}$

d) $f'(2)$ $\boxed{\text{DNE}}$ (SHARP POINT)

e) $f'(3) = \frac{5-1}{4-2} = \frac{4}{2} = \boxed{2}$

5. (2 points) In the online version of the textbook, do problem 131 in section 3.3.

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

X	1	3	4
f(x)	3	1	2
f'(x)	-1	DNE	1
g(x)	1	2.5	2.5
g'(x)	1	0	0

a) $h'(1) = (-1)(1) + (3)(1) = \boxed{2}$

b) $h'(3)$ $\boxed{\text{DNE}}$ BECAUSE $f'(3)$ DNE

c) $h'(4) = (1)(2.5) + (2)(0) = \boxed{2.5}$