

# Math 131 - Homework 4

March 31, 2021

Name key

Score \_\_\_\_\_

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 14.

1. (2 points) Use logarithmic differentiation to find  $\frac{dy}{dx}$ :  $y = \frac{x+11}{\sqrt[3]{x^2-4}}$

$$\ln y = \ln \frac{x+11}{(x^2-4)^{1/3}} = \ln(x+11) - \frac{1}{3} \ln(x^2-4)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x+11} - \frac{1}{3} \frac{2x}{x^2-4}$$

$$\frac{dy}{dx} = \left( \frac{x+11}{\sqrt[3]{x^2-4}} \right) \left( \frac{1}{x+11} - \frac{1}{3} \frac{2x}{x^2-4} \right)$$

2. (2 points) Compute  $\frac{dy}{dt}$  at  $x=1$  if  $y = x^2 + 3$  and  $\frac{dx}{dt} = 4$ .

$$\frac{dy}{dt} = 2x \frac{dx}{dt}$$

When  $x=1 \dots$

$$\frac{dy}{dt} = 2(1)(4) = \boxed{8}$$

Turn over.

3. (2 points) Find the linearization of  $f(x) = \frac{1}{x}$  at  $x = 2$ .

$$f(a) = \frac{1}{2}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f'(a) = -\frac{1}{4}$$

$$L(x) = \frac{1}{2} - \frac{1}{4}(x-2)$$

4. (2 points) Determine the differential  $dy$ .

(a)  $y = x \cos x$

$$dy = (\cos x - x \sin x) dx$$

PRODUCT  
RULE

(b)  $y = \frac{x^2 + 2}{x - 1}$

$$dy = \frac{(x-1)(2x) - (x^2+2)(1)}{(x-1)^2} dx$$

$$dy = \frac{x^2 - 2x - 2}{(x-1)^2} dx$$

QUOTIENT  
RULE

5. (2 points) Find the absolute extreme values of  $f(x) = x + \sin x$  on  $[0, 2\pi]$ .

$$f'(x) = 1 + \cos x = 0$$

↓

$$x = \pi$$

$$f(\pi) = \pi$$

Crit pt.:  $x = \pi$

END pts:  $x = 0, 2\pi$

$$f(0) = 0 \leftarrow \text{Abs min.}$$

$$f(2\pi) = 2\pi \leftarrow \text{Abs max.}$$