Math 131 - Quiz 2

January 27, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due on February 3.

1. (6 points) Evaluate each limit analytically. DO NOT USE A CALCULATOR.

(a) $\lim_{x\to4} \frac{(x^2-3x-4)^2}{x-4}$

0/0 MORE WORK

$= \lim_{x\to4} \frac{(x-4)^2(x+1)^2}{x-4} = \lim_{x\to4} (x-4)(x+1)^2$

$= (0)(25) = \boxed{0}$

x^2-3x-4

$= (x-4)(x+1)$

(b) $\lim_{\theta\to\pi/3} \frac{4\sin 2\theta}{5\theta} = \frac{4\sin\frac{2\pi}{3}}{5\pi/3} = \frac{4\left(\frac{\sqrt3}{2}\right)}{5\pi/3} = \frac{6\sqrt3}{5\pi}$

(c) $\lim_{x\to3} \frac{3x-9}{\sqrt{3x}-3}$

0/0 MORE WORK

$\cdot \frac{\sqrt{3x}+3}{\sqrt{3x}+3} = \lim_{x\to3} \frac{(\cancel{3x-9})(\sqrt{3x}+3)}{\cancel{3x-9}} = \sqrt9 + 3$

$= \boxed{6}$

(d) $\lim_{u\to-4} \left(\frac{4u}{2u+8} + \frac{8}{u+4}\right)$

DIVISION BY ZERO. MORE WORK

$= \lim_{u\to-4} \left(\frac{2u}{u+4} + \frac{8}{u+4}\right) = \lim_{u\to-4} \frac{2u+8}{u+4}$

$= \lim_{u\to-4} \frac{2(\cancel{u+4})}{\cancel{u+4}} = \boxed{2}$

Turn over.

2. (2 points) Consider the following limit: $\lim_{x \to 5} \frac{x^2 - 10x + 20}{(x-5)^2}$

(a) Explain why the limit laws cannot be used to determine the limit.

THE LIMIT LAWS CANNOT BE USED BECAUSE THE LIMIT OF THE DENOMINATOR IS ZERO.

(b) The actual limit fails to exist. In which of the four common ways does it fail to exist? Justify your reasoning.

DIRECT SUBSTITUTION YIELDS THE FORM $\frac{-5}{0}$.

THIS FORM INDICATES THAT THE FUNCTION VALUES GROW WITHOUT BOUND AS $X \to 5$. IN FACT,

$$\lim_{X \to 5^-} \frac{X^2 - 10X + 20}{(X-5)^2} = \lim_{X \to 5^+} \frac{X^2 - 10X + 20}{(X+5)^2} = -\infty.$$

3. (2 points) Evaluate each limit analytically. DO NOT USE A CALCULATOR.

(a) $\lim_{x \to 1^+} (5x^2 - x + \sqrt{4x})$

$$= 5(1)^2 - 1 + \sqrt{4(1)} = 5 - 1 + 2 = \boxed{6}$$

∅ MORE WORK

(b) $\lim_{y \to 4^-} \frac{4y - y^2}{|y-4|}$

$y < 4$

$\Downarrow$

$|y-4| = -(y-4)$

$$= \lim_{y \to 4^-} \frac{y(4-y)}{-(y-4)} = \lim_{y \to 4^-} \frac{y\cancel{(4-y)}}{\cancel{4-y}} = \boxed{4}$$