Math 131 - Quiz 3

February 24, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 3.

1. (4 points) Use the appropriate differentiation rules to determine each derivative.

(a) $\frac{d}{ds}(6s^4 - 8s^3 + 13s)$ = $24s^3 - 24s^2 + 13$

(b) $\frac{d}{dx}(5\tan x + x^2 \sin x)$ = $5\sec^2 x + 2x\sin x + x^2\cos x$

PRODUCT RULE

(c) $\frac{d}{dx}\left(\frac{x^2+2x+3}{x^2+9}\right)$ = $\frac{(x^2+9)(2x+2) - (x^2+2x+3)(2x)}{(x^2+9)^2}$

$= \frac{-2x^2+12x+18}{(x^2+9)^2}$

(d) $\frac{d}{dw}\sqrt[3]{w^7+1}$

$= \frac{d}{dw}(w^7+1)^{1/3}$

$= \frac{1}{3}(w^7+1)^{-2/3}(7w^6) = \frac{7w^6}{3\sqrt[3]{(w^7+1)^2}}$

Turn over.

2. (2 points) Find an equation of the line tangent to the graph of $f(x) = \sin^2 x$ at the point where $x = \pi/6$.

$f(x) = (\sin x)^2$

$f'(x) = 2(\sin x)^1 \cos x$

Point: $x = \frac{\pi}{6}$

$y = f(\frac{\pi}{6}) = (\sin \frac{\pi}{6})^2$

$= \frac{1}{4}$

Slope:

$m = f'(\frac{\pi}{6}) = 2\sin \frac{\pi}{6} \cos \frac{\pi}{6}$

$= 2(\frac{1}{2})(\frac{\sqrt{3}}{2})$

$= \sqrt{3}/2$

Tangent line:

$y - \frac{1}{4} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

3. (2 points) Find $g''(x)$ if $g(x) = \cot x$.

$g'(x) = -\csc^2 x$

$g''(x) = (-2\csc x)(-\csc x \cot x)$

Chain rule

$= 2\csc^2 x \cot x$

4. (2 points) Suppose that y is implicitly defined as a function of x by the equation $x^5 + 5xy + y^5 = x$. Use implicit differentiation to find $\frac{dy}{dx}$.

$\frac{d}{dx}(x^5 + 5xy + y^5) = \frac{d}{dx}(x)$

$5x^4 + 5\frac{d}{dx}(xy) + \frac{d}{dx}(y^5) = 1$

$5x^4 + 5\left[y + x\frac{dy}{dx}\right] + 5y^4\frac{dy}{dx} = 1$

$5x^4 + 5y + 5x\frac{dy}{dx} + 5y^4\frac{dy}{dx} = 1$

$(5x + 5y^4)\frac{dy}{dx} = 1 - 5x^4 - 5y$

$\frac{dy}{dx} = \frac{1 - 5x^4 - 5y}{5x + 5y^4}$