Math 131 - Quiz 4
March 24, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 31.

1. (4 points) Use the appropriate differentiation rules to determine each derivative. Do not simplify, but show all work.

(a) $\frac{d}{dx}(x^2e^{-2x})$

$$= 2xe^{-2x} + x^2\left(e^{-2x}\right)(-2) = \boxed{\left(2x - 2x^2\right)e^{-2x}}$$

(b) $\frac{d}{dx}\sin^{-1}(e^x)$

$$= \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = \boxed{\frac{e^x}{\sqrt{1-e^{2x}}}}$$

(c) $\frac{d}{dx}\ln[(5x+1)^3]$

$$= \frac{d}{dx} 3\ln(5x+1) = \frac{3}{5x+1} \cdot 5 = \boxed{\frac{15}{5x+1}}$$

(d) $\frac{d}{dt}5^{t^2}$

$$= \frac{d}{dt} e^{t^2 \ln 5} = e^{t^2 \ln 5} \cdot 2t \ln 5$$

$$= \boxed{5^{t^2} \cdot 2t \ln 5}$$

Turn over.

2. (2 points) Let $f(x) = x^3 + 5x + 1$. Find $(f^{-1})'(-5)$.

$$(f^{-1})'(-5) = \frac{1}{f'(f^{-1}(-5))} = \boxed{\frac{1}{8}}$$

$$f^{-1}(-5) = y$$

$$\Updownarrow$$

$$y^3 + 5y + 1 = -5 \Rightarrow y = -1$$

$$f'(x) = 3x^2 + 5$$

$$f'(f^{-1}(5)) = 3(-1)^2 + 5 = 8$$

3. (2 points) Compute $\frac{dx}{dt}$ at $x = -2$ if $y = 2x^2 + 1$ and $\frac{dy}{dt} = -1$.

$$y = 2x^2 + 1$$

$$\frac{dy}{dt} = 4x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{4x}\frac{dy}{dt}$$

When $x = -2$...

$$\frac{dx}{dt} = \frac{1}{4(-2)}(-1) = \boxed{\frac{1}{8}}$$

4. (2 points) Find the linearization of $f(x) = e^x$ at $x = 0$. Then use your linearization to approximate $e^{0.05}$.

$$f'(x) = e^x$$

$$f'(0) = e^0 = 1$$

$$f(0) = e^0 = 1$$

$$L(x) = f(0) + f'(0)(x - 0)$$

$$\boxed{L(x) = 1 + x}$$

$$\boxed{e^{0.05} \approx 1 + 0.05 = 1.05}$$