Math 131 - Quiz 5

April 21, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 28.

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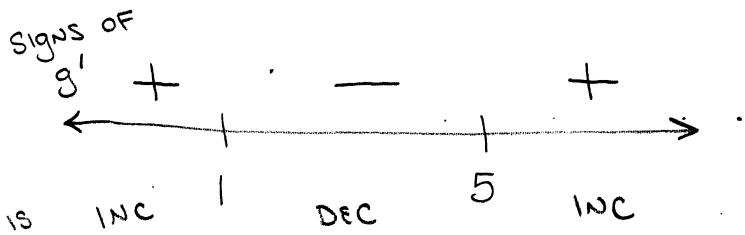
1. (6 points) Let $g(x) = x^3 - 9x^2 + 15x + 3$. (a) Find open intervals on which g is increasing/decreasing. (b) Identify all relative extrema. (c) Find open intervals on which the graph of g is concave up/down. (d) Identify all inflection points.

$g'(x) = 3x^2 - 18x + 15$

$= 3(x-5)(x-1)$

$g'(x) = 0 \Rightarrow x = 5, \; x = 1$

$g'(x)$ DNE never



(a) g IS INCREASING ON $(-\infty, 1) \cup (5, \infty)$

g IS DECREASING ON $(1, 5)$

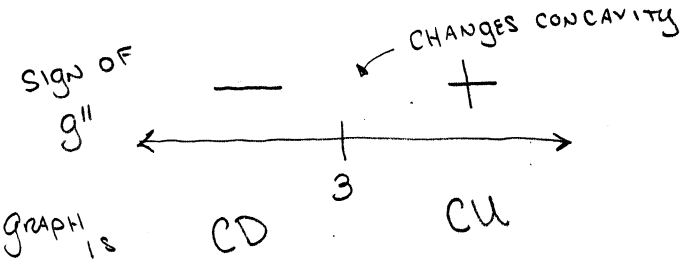
(b) $g(1) = 10$ IS A RELATIVE MAX

$g(5) = -22$ IS A RELATIVE MIN

$g''(x) = 6x - 18$

$g''(x) = 0 \Rightarrow x = 3$

$g''(x)$ DNE never



(c) GRAPH OF g IS CONCAVE DOWN ON $(-\infty, 3)$.

GRAPH IS CONCAVE UP ON $(3, \infty)$.

(d) $(3, -6)$ IS AN INFLECTION PT.

2. (2 points) Compute the limit. Show work.

$$\lim_{x\to-\infty}\left(\frac{4x^5-7x^3+13}{17x^5-1}\right)\cdot\frac{\frac{1}{x^5}}{\frac{1}{x^5}}$$

$$\lim_{x\to-\infty}\frac{4-\frac{7}{x^2}+\frac{13}{x^5}}{17-\frac{1}{x^5}}=\frac{4-0+0}{17-0}$$

$$=\boxed{\frac{4}{17}}$$

3. (2 points) Find the horizontal asymptote(s) of the graph of $y=\dfrac{|x|^3}{7x^3+5x}$.

$$\lim_{x\to\infty}\frac{|x|^3}{7x^3+5x}=\lim_{x\to\infty}\frac{x^3}{7x^3+5x}=\lim_{x\to\infty}\frac{1}{7+\frac{5}{x^2}}=\frac{1}{7}$$

For $x>0$, $|x|=x$

$$\lim_{x\to-\infty}\frac{|x|^3}{7x^3+5x}=\lim_{x\to-\infty}\frac{-x^3}{7x^3+5x}=\lim_{x\to\infty}\frac{-1}{7+\frac{5}{x^2}}=-\frac{1}{7}$$

For $x<0$, $|x|=-x$

Horizontal Asymptotes: $\boxed{y=\frac{1}{7},\ y=-\frac{1}{7}}$