Math 131 - Test 1

February 10, 2021

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist). When classifying discontinuities, use the words *removable*, *nonremovable*, *infinite*, or *jump*.

1. (6 points) Think about the following limit:

$$\lim_{x \to 2} (x^2 + 1) = 5.$$

In one or two sentences, carefully explain what this expression means. (Tell what it means, not how to get it.)

IT MEANS ... AS THE VALUES OF X GET CLOSER & CLOSER TO 2 FROM EITHER SIDE, THE VALUES OF X^2+1 GET CLOSER & CLOSER TO 5.

2. (9 points) Let $f(x) = \dfrac{2x^2 + 3x}{|x|}$.

(a) Use a table of numerical values to find (estimate) $\lim_{x \to 0^+} f(x)$.

X	0.1	0.01	0.001	0.0001
f(x)	3.2	3.02	3.002	3.0002

IT LOOKS LIKE $\lim_{x \to 0^+} f(x) = 3$

(b) Determine the limit analytically: $\lim_{x \to 0^-} f(x)$

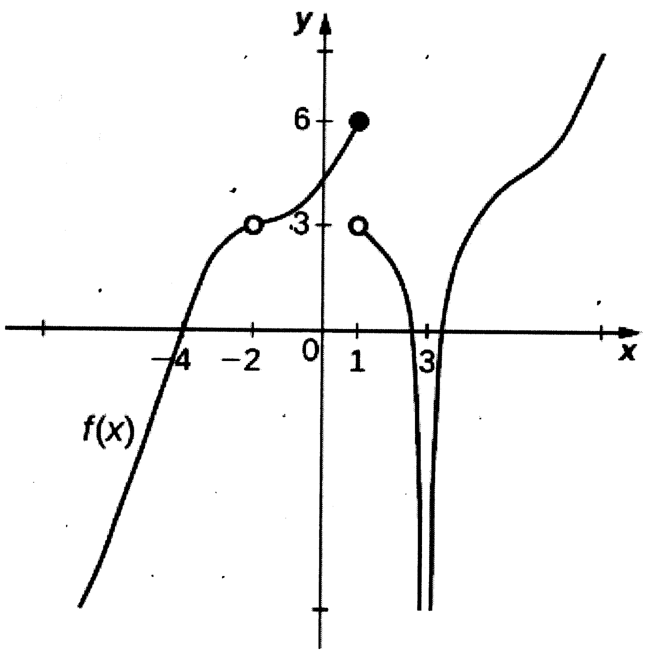
0/0 MORE WORK

$$\lim_{x \to 0^-} \frac{2x^2+3x}{|x|} = \underbrace{\lim_{x \to 0^-}}_{|x| = -x \text{ for } x<0} \frac{x(2x+3)}{-x} = \lim_{x \to 0^-} -(2x+3) = \boxed{-3}$$

(c) What do your results from above tell you about $\lim_{x \to 0} f(x)$? Briefly explain.

$$\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x) \Rightarrow \boxed{\lim_{x \to 0} f(x) \text{ DNE}}$$

3. (10 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -4} f(x) = \boxed{0}$

(b) $\lim_{x \to 1} f(x)$ $\boxed{\text{DNE}}$ BECAUSE $\lim_{x \to 1^-} f(x) = 6$ AND $\lim_{x \to 1^+} f(x) = 3$

(c) $\lim_{x \to 3} f(x) = \boxed{-\infty}$

(d) $\lim_{x \to 1^+} f(x) = \boxed{3}$

(e) $\lim_{x \to -2^-} f(x) = \boxed{3}$

4. (3 points) Using the graph above, find and classify the discontinuities of f.

DISCONT. AT $X = -2$ IS REMOVABLE.

DISCONT AT $X = 1$ IS jump/NONREMOVABLE.

DISCONT AT $X = 3$ IS INFINITE/NONREMOVABLE

5. (12 points) Use the limit laws and the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to determine each of the following. Show your work.

(a) $\lim_{x\to 0} \frac{2\sin x \cos 8x}{5x}$ $= \frac{2}{5}\left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \cos 8x\right) = \frac{2}{5}(1)(\cos 0)$

$= \boxed{\frac{2}{5}}$

(b) $\lim_{x\to 0} \frac{\tan x}{x}$ $= \lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{1}{\cos x} = \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{\cos x}\right) = (1)(1)$

$= \boxed{1}$

(c) $\lim_{x\to 0} \frac{x}{\tan x}$ $= \lim_{x\to 0}\underbrace{\left(\frac{1}{\frac{\tan x}{x}}\right)}_{\text{From part (b)}} = \frac{1}{1} = \boxed{1}$

IN OUR NOTES, THIS IS...

6. (12 points) These limits DO NOT EXIST. **Choose any three (3) of them,** and clearly tell why the limit fails to exist. If necessary, provide evidence.

#4 (a) $\lim_{x\to 4} \sqrt{x-4}$

$\sqrt{x-4}$ IS NOT DEFINED ON AN OPEN INTERVAL AROUND $x=4$.

#2 (b) $\lim_{x\to 1} \frac{1}{(x-1)^2}$

FORM $1/0 \Rightarrow$ FUNCTION VALUES GROW WITHOUT BOUND AS LIMIT POINT IS APPROACHED.

#3 (c) $\lim_{x\to 0} \sin\left(\frac{1}{x^2}\right)$

VALUES OF $\sin\left(\frac{1}{x^2}\right)$ OSCILLATE WILDLY ON ANY INTERVAL AROUND $x=0$.

(d) $\lim_{x\to 2} g(x)$, where $g(x) = \begin{cases} 6x+\sin(\pi x), & x<2 \\ 6x+\pi x, & x>2 \end{cases}$

#1

$\lim_{x\to 2^-} g(x) = 6(2) + \sin 2\pi = 12$

$\neq \lim_{x\to 2^+} g(x) = 6(2) + 2\pi$

$= 12 + 2\pi$

7. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to -2} \left(\frac{3x}{x+2} + \frac{6}{x+2} \right)$ $= \lim_{x \to -2} \frac{3x+6}{x+2} = \lim_{x \to -2} \frac{3\cancel{(x+2)}}{\cancel{x+2}} = \boxed{3}$

ZERO DENOM

(b) $\lim_{x \to 1} \frac{x^2+4x-5}{x^2+x-2}$ $= \lim_{x \to 1} \frac{(x+5)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x+5}{x+2} = \frac{6}{3} = \boxed{2}$

0/0

(c) $\lim_{w \to 0} \frac{w}{(w+4)^2 - 16}$ $= \lim_{w \to 0} \frac{w}{w^2+8w+16-16} = \lim_{w \to 0} \frac{\cancel{w}}{\cancel{w}(w+8)} = \boxed{\frac{1}{8}}$

0/0

(d) $\lim_{r \to 2^+} \left(\frac{r^2+7}{\cos \pi r} \right)$ $= \frac{(2)^2+7}{\cos 2\pi} = \frac{11}{1} = \boxed{11}$

8. (9 points) Each function given below has a single point of discontinuity. Find the point of discontinuity and classify the discontinuity. Explain your reasoning.

(a) $g(x) = \frac{1}{x} + 1$

DISCONT AT $X=0$

IS AN INFINITE DISCONT.

$\lim_{X\to 0} \frac{1}{X}$ HAS $\frac{\text{NONZERO}}{\text{ZERO}}$ FORM.

(b) $h(x) = \frac{|x+1|}{x+1}$

DISCONT AT $X=-1$ IS A jump DISCONT.

$\lim_{X\to 1^-} h(x) = -1 \neq 1 = \lim_{X\to 1^+} h(x)$

(c) $f(x) = \frac{x^2-4}{x-2}$

DISCONT AT $X=2$ IS REMOVABLE.

$\lim_{X\to 2} f(x) = \lim_{X\to 2} \frac{\cancel{(X-2)}(X+2)}{\cancel{X-2}} = 4.$ LIMIT EXISTS!

9. (4 points) Use limits to explain what it means for a function f to be continuous at $x = 2$.

f IS CONT. AT $X=2$ IFF $\boxed{\lim_{X\to 2} f(x) = f(2)}$

10. (6 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\displaystyle\lim_{x\to 3} \frac{x}{(x-3)^2}$ $\quad 3/0 \Rightarrow$ SOME KIND OF INF LIMITS

LEFT OF $X=3$,

$\frac{X}{(X-3)^2}$ HAS FORM $\frac{\text{POS}}{\text{POS}} \Rightarrow \displaystyle\lim_{X\to 3^-} \frac{X}{(X-3)^2} = +\infty$

RIGHT OF $X=3$,

$\frac{X}{(X-3)^2}$ HAS FORM $\frac{\text{POS}}{\text{POS}} \Rightarrow \displaystyle\lim_{X\to 3^+} \frac{X}{(X-3)^2} = +\infty$

$\displaystyle\lim_{X\to 3} \frac{X}{(X-3)^2} = +\infty$

(b) $\displaystyle\lim_{x\to 7^-} \left(\frac{x}{x-7}\right)$ $\quad 7/0 \Rightarrow$ SOME KIND OF INF LIMIT

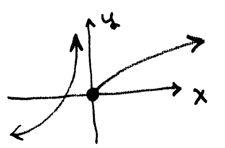
LEFT OF $X=7$,

$\frac{X}{X-7}$ HAS FORM $\frac{\text{POS}}{\text{NEG}} \Rightarrow$ $\displaystyle\lim_{X\to 7^-} \frac{X}{X-7} = -\infty$

11. (5 points) Determine whether each statement is true (T) or false (F).

(a) __F__ If f has a limit at $x=2$, then f must be defined at $x=2$. $\quad$ EX $\displaystyle\lim_{X\to 2} \frac{\sin(X-2)}{X-2} = 1$

(b) __F__ If the graph of g has the vertical asymptote $x=0$, then $g(0)$ is not defined.



(c) __F__ If f has a removable discontinuity at $x=5$, then the limit at $x=5$ does not exist.

BY DEFINITION, LIMIT MUST EXIST.

(d) __F__ If $\displaystyle\lim_{x\to 1^+} f(x) = f(1)$, then f is continuous at $x=1$.

$\displaystyle\lim_{X\to 1} f(x) = f(1) \Rightarrow$ CONTINUITY AT $X=1$

(e) __T__ The limit of a polynomial function can always be found by direct substitution.

PLUG IN!