Math 131 - Test 1

February 10, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist). When classifying discontinuities, use the words *removable*, *nonremovable*, *infinite*, or *jump*.

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1. (6 points) Think about the following limit:

$$\lim_{x \to 2} (x^2 + 1) = 5.$$

In one or two sentences, carefully explain what this expression means. (Tell what it means, not how to get it.)

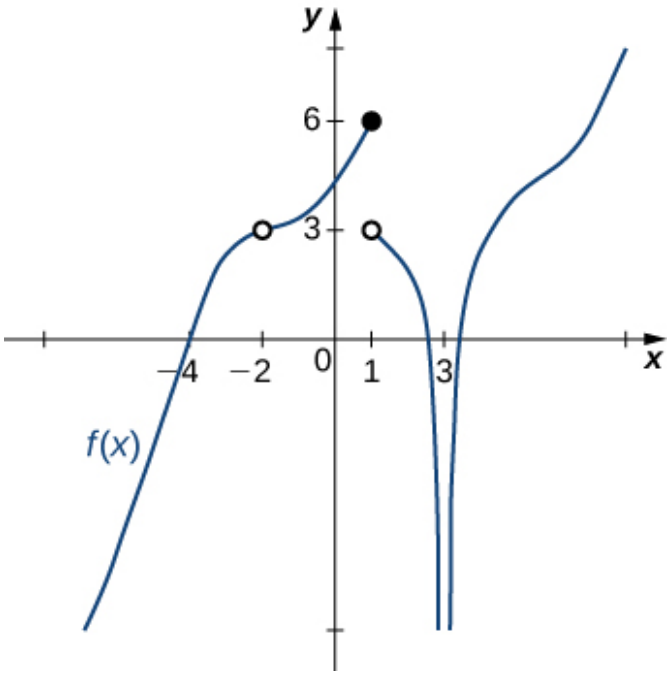
2. (9 points) Let $f(x) = \dfrac{2x^2 + 3x}{|x|}$.

(a) Use a table of numerical values to find (estimate) $\lim_{x \to 0^+} f(x)$.

(b) Determine the limit analytically: $\lim_{x \to 0^-} f(x)$

(c) What do your results from above tell you about $\lim_{x \to 0} f(x)$? Briefly explain.

3. (10 points) Referring to the graph of $y = f(x)$ shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to -4} f(x)$

(b) $\lim_{x \to 1} f(x)$

(c) $\lim_{x \to 3} f(x)$

(d) $\lim_{x \to 1^+} f(x)$

(e) $\lim_{x \to -2^-} f(x)$

4. (3 points) Using the graph above, find and classify the discontinuities of f.

5. (12 points) Use the limit laws and the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to determine each of the following. Show your work.

   (a) $\lim_{x\to 0} \frac{2\sin x \cos 8x}{5x}$

   (b) $\lim_{x\to 0} \frac{\tan x}{x}$

   (c) $\lim_{x\to 0} \frac{x}{\tan x}$

6. (12 points) These limits DO NOT EXIST. **Choose any three (3) of them**, and clearly tell why the limit fails to exist. If necessary, provide evidence.

   (a) $\lim_{x\to 4} \sqrt{x-4}$

   (b) $\lim_{x\to 1} \frac{1}{(x-1)^2}$

   (c) $\lim_{x\to 0} \sin\left(\frac{1}{x^2}\right)$

   (d) $\lim_{x\to 2} g(x)$, where $g(x) = \begin{cases} 6x + \sin(\pi x), & x < 2 \\ 6x + \pi x, & x > 2 \end{cases}$

7. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to -2} \left(\dfrac{3x}{x+2} + \dfrac{6}{x+2} \right)$

(b) $\lim_{x \to 1} \dfrac{x^2 + 4x - 5}{x^2 + x - 2}$

(c) $\lim_{w \to 0} \dfrac{w}{(w+4)^2 - 16}$

(d) $\lim_{r \to 2^+} \left(\dfrac{r^2 + 7}{\cos \pi r} \right)$

8. (9 points) Each function given below has a single point of discontinuity. Find the point of discontinuity and classify the discontinuity. Explain your reasoning.

(a) $g(x) = \dfrac{1}{x} + 1$

(b) $h(x) = \dfrac{|x+1|}{x+1}$

(c) $f(x) = \dfrac{x^2 - 4}{x - 2}$

9. (4 points) Use limits to explain what it means for a function f to be continuous at $x = 2$.

10. (6 points) In each problem below, determine whether the limit is $+\infty$, $-\infty$, or DNE. Show work or explain your reasoning.

(a) $\lim_{x \to 3} \dfrac{x}{(x-3)^2}$

(b) $\lim_{x \to 7^-} \left(\dfrac{x}{x-7} \right)$

11. (5 points) Determine whether each statement is true (T) or false (F).

(a) _____ If f has a limit at $x = 2$, then f must be defined at $x = 2$.

(b) _____ If the graph of g has the vertical asymptote $x = 0$, then $g(0)$ is not defined.

(c) _____ If f has a removable discontinuity at $x = 5$, then the limit at $x = 5$ does not exist.

(d) _____ If $\lim_{x \to 1^+} f(x) = f(1)$, then f is continuous at $x = 1$.

(e) _____ The limit of a polynomial function can always be found by direct substitution.