Math 131 - Test 3
April 14, 2021

Name key

Score ____________

Show all work to receive full credit. Supply explanations where necessary. Use differentiation rules for all derivatives. Unless otherwise indicated, do not simplify your derivatives.

1. (12 points) The graph of the equation $2x^3+2y^3-9xy=0$ is called a *folium of Descartes.*

   (a) Use implicit differentiation to find dy/dx.

$$\frac{d}{dx}\left(2x^3+2y^3-9xy\right)=\frac{d}{dx}(0)$$

$$6x^2+6y^2\frac{dy}{dx}-9y-9x\frac{dy}{dx}=0$$

$$\left(6y^2-9x\right)\frac{dy}{dx}=-6x^2+9y \Rightarrow \boxed{\frac{dy}{dx}=\frac{-6x^2+9y}{6y^2-9x}}$$

   (b) Find an equation of the line tangent to the graph at the point $(2,1)$.

$$m=\left.\frac{dy}{dx}\right|_{(2,1)}=\frac{-6(4)+9}{6-18}=\frac{-15}{-12}=\frac{5}{4}$$

$$\boxed{\text{TAN. LINE IS } \quad y-1=\frac{5}{4}(x-2).}$$

   (c) Find an equation of the line normal to the graph at the point $(2,1)$.

$$m_\perp=-\frac{4}{5}$$

$$\boxed{\text{NORMAL LINE IS } \quad y-1=-\frac{4}{5}(x-2)}$$

2. (5 points) Think about the graph of $y = \cos^{-1}(x^2)$. Find the slope of the tangent line at the point where $x = 1/2$. Round your answer to four places.

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot (2x) = \frac{-2x}{\sqrt{1-x^4}}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = \frac{-2(\frac{1}{2})}{\sqrt{1-(\frac{1}{2})^4}} = \frac{-1}{\sqrt{1-\frac{1}{16}}} \approx -1.0328$$

3. (7 points) Let $f(x) = x^3 + 2x + 3$. Compute $(f^{-1})'(0)$.

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(-1)} = \frac{1}{3(-1)^2+2} = \boxed{\frac{1}{5}}$$

$$f^{-1}(0) = w \Leftrightarrow w^3 + 2w + 3 = 0 \qquad f'(x) = 3x^2 + 2$$

$$\Rightarrow w = -1$$

$$\Rightarrow f^{-1}(0) = -1$$

4. (6 points) Use logarithmic differentiation to find dy/dx if $y = x^{\ln x}$.

$$\ln y = \ln x^{\ln x}$$

$$\ln y = \ln x \cdot \ln x$$

$$\ln y = (\ln x)^2$$

$$\frac{1}{y}\frac{dy}{dx} = 2(\ln x)^1\left(\frac{1}{x}\right)$$

$$\boxed{\frac{dy}{dx} = x^{\ln x} \cdot \frac{2\ln x}{x}}$$

$$\frac{d}{dx}a^x = a^x \ln a$$

5. (15 points) Differentiate. Do not simplify.

(a) $\frac{d}{dx}(2^{4x} + 4x^2)$

$$= \frac{d}{dx}2^{4x} + \frac{d}{dx}4x^2$$

$$= (2^{4x})(4\ln 2) + 8x$$

(b) $\frac{d}{dt}\log_7(6t^4+3)^5$

CHANGE-OF-BASE FORMULA.

$$= \frac{d}{dt}\frac{5\ln(6t^4+3)}{\ln 7} = \frac{5}{\ln 7}\frac{24t^3}{6t^4+3}$$

(c) $\frac{d}{dx}(1+\tan^{-1}x)^3$

$$= 3(1+\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right)$$

$$= \frac{3(1+\tan^{-1}x)^2}{1+x^2}$$

6. (8 points) A particle is moving along the circle $x^2+y^2 = 25$. At the point in the 1st quadrant where $x = 3$, $\frac{dy}{dt} = -7$. Find $\frac{dx}{dt}$ at that point.

$$x^2+y^2 = 25 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-2y\frac{dy}{dt}}{2x}$$

When $x = 3$, $y^2 = 16 \Rightarrow y = 4$

$$\therefore \frac{dx}{dt} = \frac{-2(4)(-7)}{2(3)} = \frac{28}{3}$$

7. (8 points) Find the linearization of $f(x) = \dfrac{1}{x}$ at $x = 3$. Then use your linearization to approximate $\dfrac{1}{2.97}$.

$$f'(x) = -\frac{1}{x^2}$$

$$f'(3) = -\frac{1}{9}$$

$$f(3) = \frac{1}{3}$$

$$\boxed{L(x) = \frac{1}{3} - \frac{1}{9}(x-3)}$$

$$\frac{1}{2.97} \approx L(2.97) = \frac{1}{3} - \frac{1}{9}(-0.03)$$

$$= \frac{1}{3} + \frac{0.01}{3} = \frac{1.01}{3}$$

$$= \boxed{0.33\overline{6}}$$

8. (5 points) Find the differential dy if $y = \dfrac{x^2+2}{x-1}$.

$$dy = \frac{2x(x-1) - (x^2+2)(1)}{(x-1)^2}\,dx$$

$$\boxed{dy = \frac{x^2 - 2x - 2}{(x-1)^2}\,dx}$$

9. (6 points) Find the critical points: $y = \sqrt{4 - x^2}$.

DOMAIN IS $[-2, 2]$.

$$\frac{dy}{dx} = \frac{1}{2}(4-x^2)^{-1/2}(-2x)$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$\boxed{X = 0 \text{ IS THE ONLY CRITICAL POINT.}}$

$\frac{dy}{dx} = 0$ WHEN $X = 0$.

$\frac{dy}{dx}$ DNE WHEN $X = \pm 2$. ← NOT DOMAIN INTERIOR PTS!

10. (10 points) Find the absolute extreme values of $g(x) = 3x^4 - 8x^3 - 48x^2 + 5$ on $[-3, 1]$.

$g'(x) = 12x^3 - 24x^2 - 96x$

$= 12x(x^2 - 2x - 8)$

$= 12x(x-4)(x+2) = 0$

$x = 0$, $\cancel{x = 4}$, $x = -2$

CRIT PTS ARE $x = 0$, $x = -2$

END PTS ARE $x = -3$, $x = 1$

$g(0) = 5$

$g(-2) = -75 \leftarrow$ ABS MIN

$g(-3) = 32 \leftarrow$ ABS MAX

$g(1) = -48$

11. (5 points) Some values of $f(x)$ and $f'(x)$ are given in the table below. Use the table to find the linearization of f at $x = 1.25$

x	0.50	0.75	1.00	1.25	1.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

$L(x) = f(1.25) + f'(1.25)(x - 1.25)$

$L(x) = 9.41 + 6.26(x - 1.25)$

OR

$L(x) = 6.26x + 1.585$

12. (3 points) Is it possible for a function to have more than one absolute maximum value? Explain.

No, AN ABSOLUTE MAX IS THE SINGLE BIGGEST FUNCTION VALUE, IF THERE IS ONE.

AN ABS MAX MAY OCCUR AT SEVERAL PLACES, BUT THERE'S ONLY A SINGLE MAX Y-VALUE.

13. (3 points) Is it possible for a function to have no absolute maximum value? Explain.

YES, FOR EXAMPLE, $f(x) = x^2$.

HOWEVER, CONTINUOUS FUNCTIONS ON CLOSED & BOUNDED INTERVALS MUST HAVE A MAX.

14. (4 points) Explain what it means to be a critical point for a function.

A CRIT. PT. OF f IS A DOMAIN INTERIOR POINT WHERE f' IS ZERO OR DOES NOT EXIST.

15. (3 points) When looking at the graph of a function, how would you identify any critical points for the function?

LOOK FOR DOMAIN INTERIOR POINTS WHERE THE TANGENT LINE IS HORIZONTAL OR VERTICAL OR THE GRAPH HAS SHARP POINTS.