

Math 131 - Final Exam

May 12, 2021

Name _____

Score _____

Show all work to receive full credit. For each problem, place your final answer in the box provided. Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation.

1. Determine the limit. Use algebraic techniques (not a graph, table, or L'Hôpital's rule) to show how you got your answer.

$$\lim_{x \rightarrow 4} \left(\frac{\sqrt{x+5} - 3}{x-4} \right)$$

2. Determine the limit. Show analytically (not with a graph or table) how you got your answer.

$$\lim_{x \rightarrow 1^-} \left(\frac{2x-2}{x^2-2x+1} \right)$$

3. Yes or No: Is g continuous at $x = 0$? Use the definition of continuity to support your answer.

$$g(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

4. Let $f(x) = x - x^2$. Write $f'(x)$ in the box, then use the limit definition of derivative to obtain your answer.

5. Let $f(x) = \frac{\tan x}{2e^x}$. Compute $f'(0)$.

6. A ball is thrown straight upward in such a way that its height (in feet) after t seconds is given by

$$h(t) = -16t^2 + 48t + 144.$$

How high is the ball one second after it reaches its maximum height?

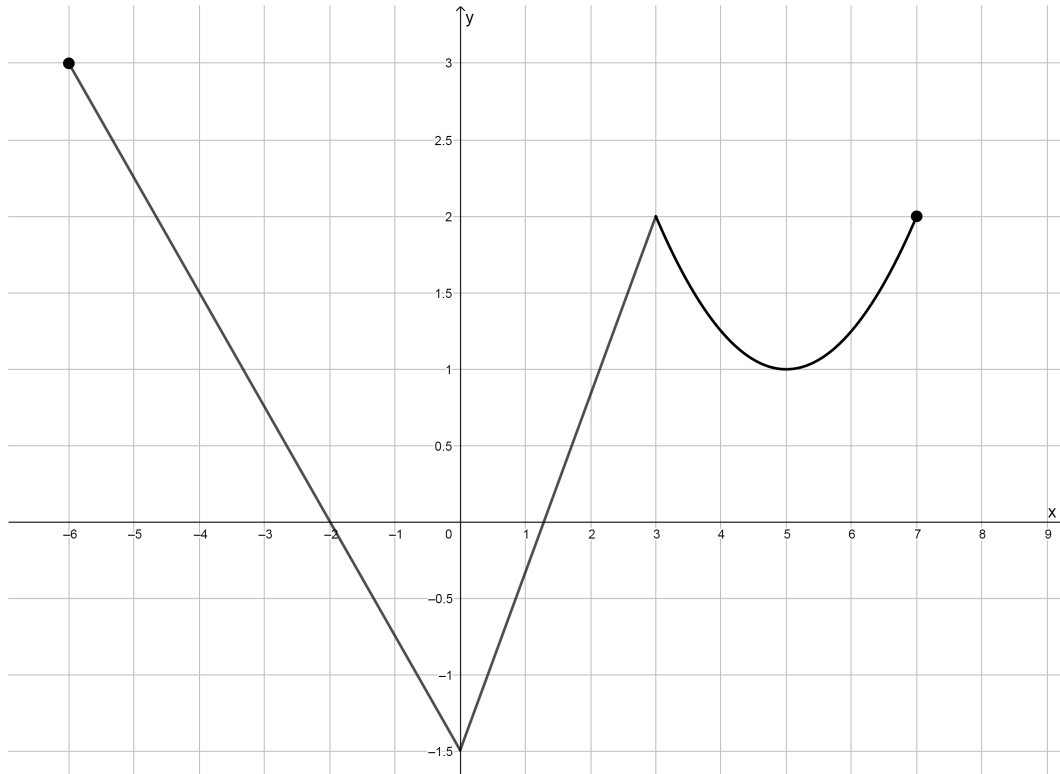
7. Find $\frac{dy}{dx}$ if $y = (x^2 + \tan^{-1} x)^3$.

8. Find an equation of the line normal to the graph of $x^3 + y^2 = xy + 3$ at the point $(1, 2)$.

9. Let $g(x) = \frac{(x+1)^2}{(x+3)^4}$. Use logarithmic differentiation to find $g'(x)$.

10. Find the linearization of $f(x) = \ln x$ at $x = 1$, and use it to approximate $\ln 1.05$.

11. The graph of $y = f(x)$ is shown below. Find each critical point of f and say why it is a critical point.



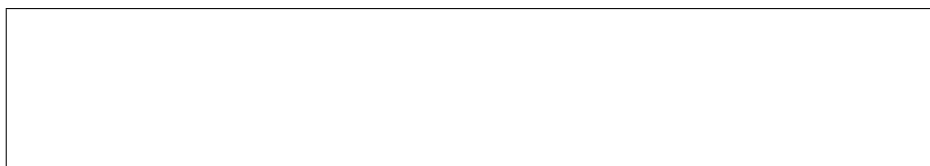
12. Use calculus techniques to find the absolute extreme values of $f(x) = x^3 - 6x^2 - 6$ on $[-1, 7]$.

13. The first derivative of f is given by $f'(x) = x^3(x - 1)(x + 3)$. Locate the relative extreme values of f .

14. Find the inflection point(s) of the graph of $f(x) = x^3 - 6x^2 - 6$.

15. Evaluate the limit: $\lim_{x \rightarrow 1} \frac{\ln x^3}{x^2 - 1}$

16. Find $f(x)$ if $f'(x) = 6x^2 + e^x - \sin x$ and $f(0) = 5$.



17. Let $f(x) = \frac{1}{x}$. Use 4 subintervals of equal length and right endpoints of the subintervals to compute a Riemann sum for f on $[1, 2]$.



18. Evaluate the definite integral.

$$\int_1^2 \left(\frac{1}{x} + 2 + x^3 \right) dx$$

19. Find the area of the region between the graph of $y = \sqrt{x}$ and the x -axis over the interval $[0, 4]$.

20. In order to evaluate the following integral, an appropriate u -substitution should be made. Carry out the substitution and write the new integral. DO NOT EVALUATE the new integral.

$$\int 5xe^{-x^2} dx$$