Math 131 - Quiz 10

April 20, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 25.

1. (6 points) The function f and its first two derivatives are shown below:

FYI: GRAPH OF f IS ATTACHED.

$$f(x) = x^{2/3}(6-x)^{1/3}, \qquad f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}, \qquad f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}.$$

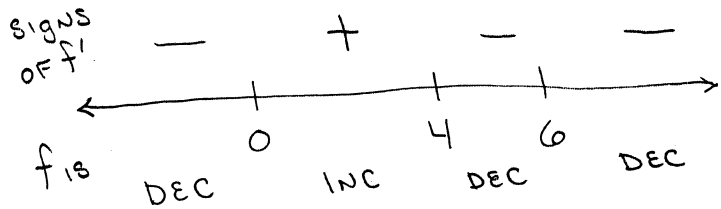
Use calculus techniques to find open intervals on which f is increasing/decreasing. Identify all relative extrema. Find open intervals on which the graph of f is concave up/down. Identify all inflection points.

1ST DERIVATIVE TEST...

$f'(x) = 0 \Rightarrow x = 4$

$f'(x)$ DNE $\Rightarrow x = 0, x = 6$

} ALL ARE CRIT #'S



f IS INCREASING ON $(0,4)$.

f IS DECREASING ON $(-\infty,0) \cup (4,6) \cup (6,\infty)$.

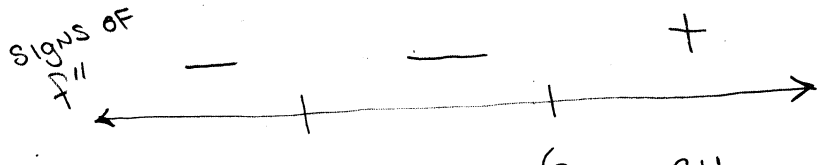
$f(0) = 0$ IS A REL. MIN.

$f(4) = \sqrt[3]{32}$ IS A REL. MAX.

2ND DERIVATIVE TEST...

$f''(x) = 0$ NEVER

$f''(x)$ DNE $\underbrace{x = 0, x = 6}_{\text{PIP's}}$



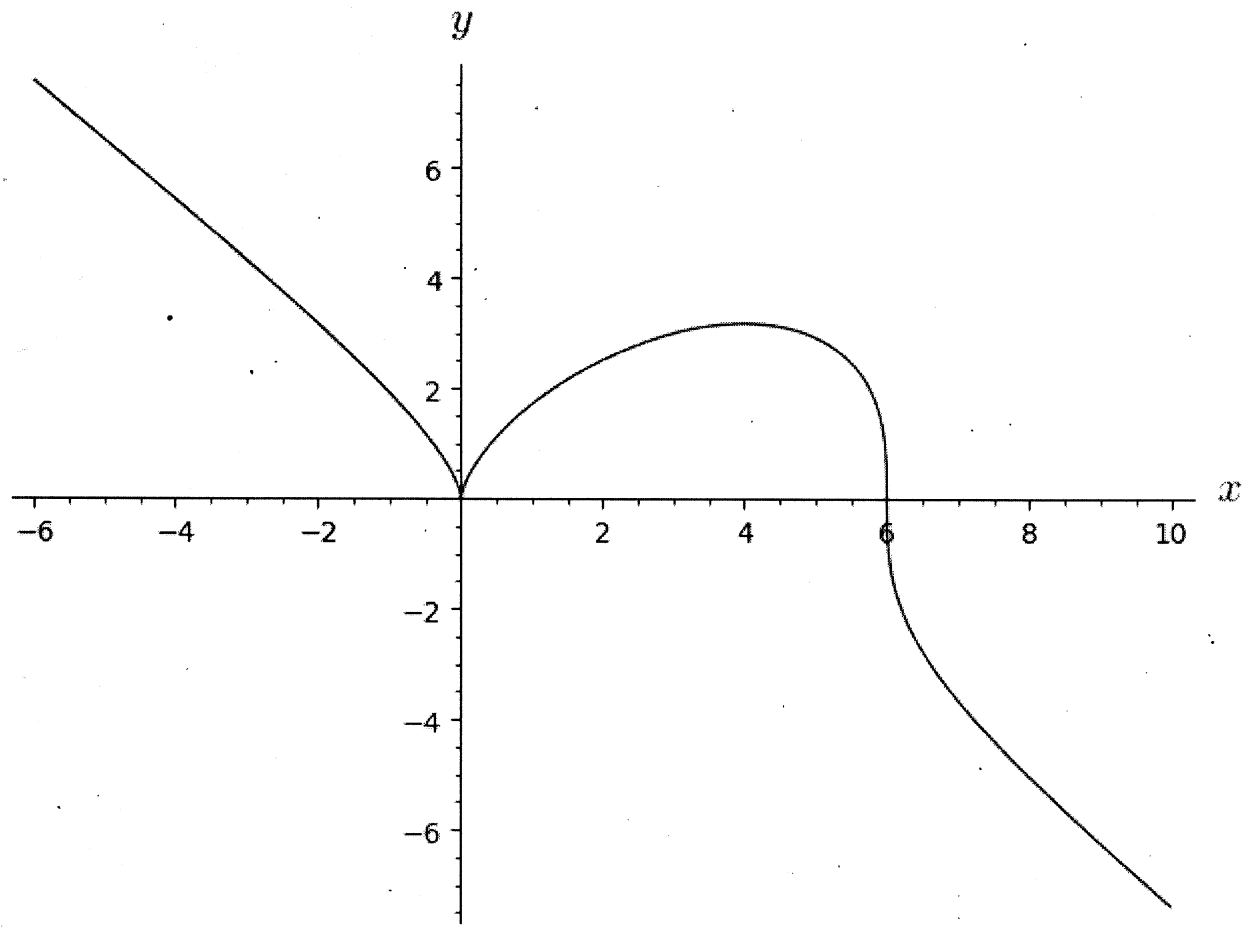
GRAPH IS CONCAVE DOWN ON $(-\infty,0) \cup (0,6)$.

GRAPH IS CONCAVE UP ON $(6,\infty)$.

$(6,0)$ IS AN INFLECTION PT.

Turn over.

Problem #1



2. (3 points) Evaluate each limit. Show all work.

(a) $\lim_{x\to\infty} \dfrac{5x^3+7x^2-8x}{2x^3-1017x^2-93} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \dfrac{5+\frac{7}{x}-\frac{8}{x^2}}{2-\frac{1017}{x}-\frac{93}{x^3}} = \dfrac{5+0-0}{2-0-0}$

$= \boxed{\dfrac{5}{2}}$

(b) $\lim_{x\to-\infty} \dfrac{x+2}{\sqrt{4x^2+1}} \cdot \dfrac{\frac{1}{-x}}{\frac{1}{\sqrt{x^2}}} = \lim_{x\to-\infty} \dfrac{-1-\frac{2}{x}}{\sqrt{4+\frac{1}{x^2}}} = \dfrac{-1-0}{\sqrt{4+0}} = \boxed{-\dfrac{1}{2}}$

Using $\sqrt{x^2} = -x$ for $x<0$

3. (1 point) Find all vertical and horizontal asymptotes of the graph of $y = \dfrac{x^2+2x}{x^3+x^2-2x}$. (You don't need to show each limit. Use shortcuts.)

$y = \dfrac{x(x+2)}{x(x+2)(x-1)} = \dfrac{1}{x-1}$; $x \neq 0$, $x \neq -2$

$\boxed{\begin{array}{l} \text{H.A. } y = 0 \\ \text{V.A. } x = 1 \end{array}}$