

# Math 131 - Quiz 11

April 28, 2022

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This quiz is due May 5.

1. (6 points) Evaluate each limit.

$$(a) \lim_{x \rightarrow \infty} \frac{x^3}{e^{x/2}} \stackrel{\infty/\infty}{=} \lim_{x \rightarrow \infty} \frac{3x^2}{\frac{1}{2}e^{x/2}} \stackrel{\infty/\infty}{=} \lim_{x \rightarrow \infty} \frac{6x}{\frac{1}{4}e^{x/2}} = \lim_{x \rightarrow \infty} \frac{6}{\frac{1}{8}e^{x/2}} = \boxed{0}$$

L'HOPITAL'S RULE THREE TIMES.

$$(b) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} \stackrel{\infty/\infty}{=} \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2}}{\sqrt{x^2+1}} = \lim_{x \rightarrow \infty} \sqrt{\frac{x^2}{x^2+1}}$$

For  $x > 0$ ,  $x = \sqrt{x^2}$

$$= \lim_{x \rightarrow \infty} \sqrt{\frac{1}{1 + \frac{1}{x^2}}} = \sqrt{1} = \boxed{1}$$

$$(c) \lim_{x \rightarrow 0^+} x^3 \cot x \stackrel{0 \cdot \infty}{=} \lim_{x \rightarrow 0^+} \frac{x^3}{\tan x} \stackrel{0/0}{=} \lim_{x \rightarrow 0^+} \frac{3x^2}{\sec^2 x} = \frac{0}{1} = \boxed{0}$$

$$(d) \lim_{x \rightarrow 1^+} \left( \frac{3}{\ln x} - \frac{2}{x-1} \right) \stackrel{\infty - \infty}{=} \lim_{x \rightarrow 1^+} \left( \frac{3x-3-2 \ln x}{(x-1) \ln x} \right) \stackrel{0/0}{=} \lim_{x \rightarrow 1^+} \left( \frac{3 - \frac{2}{x}}{\frac{x-1}{x} + \ln x} \right) \stackrel{1/0^+}{=} \boxed{+\infty}$$

Turn over.

2. (4 points) Evaluate each indefinite integral.

(a)  $\int (2x^3 + \sin x) dx$

$$= \frac{2}{4} x^4 - \cos x + C = \frac{1}{2} x^4 - \cos x + C$$

(b)  $\int (\sqrt{x} + \sqrt[5]{x^3}) dx = \int (x^{1/2} + x^{3/5}) dx$

$$= \frac{2}{3} x^{3/2} + \frac{5}{8} x^{8/5} + C$$

(c)  $\int \left( \frac{5}{x} + 3 \sec^2 x \right) dx = 5 \ln |x| + 3 \tan x + C$

(d)  $\int \left( \frac{1}{x^2+1} + \frac{1}{x^2} \right) dx = \int \frac{1}{x^2+1} dx + \int x^{-2} dx$

$$= \tan^{-1} x + (-x^{-1}) + C$$

$$= \tan^{-1} x - \frac{1}{x} + C$$