Math 131 - Quiz 4
February 16, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due February 21.

1. (4 points) Let $f(x) = x^2 - x + 3$.

   (a) Use the limit definition of the derivative to determine $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2 - (x+h) + 3] - [x^2 - x + 3]}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x - h + 3 - x^2 + x - 3}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - h}{h} = \lim_{h\to 0} (2x + h - 1) = \boxed{2x - 1}$$

   (b) Use our basic differentiation rules to determine $f'(x)$.

$$f'(x) = \frac{d}{dx} x^2 - \frac{d}{dx} x + \frac{d}{dx} 3$$

$$= 2x - 1 + 0 = \boxed{2x - 1}$$

   (c) Find an equation of the line tangent to the graph of f at the point where $x = 2$.

$m = f'(2) = 3$

POINT: $x = 2$, $y = f(2) = 5$

$y - 5 = 3(x - 2)$

OR

$y = 3x - 1$

2. (2 points) Let $g(x) = \sqrt{2x}$. Use the limit definition of the derivative to determine $g'(x)$.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}$$

$$= \lim_{h \to 0} \frac{2x + 2h - 2x}{h\left[\sqrt{2x+2h} + \sqrt{2x}\right]} = \lim_{h \to 0} \frac{2}{\sqrt{2x+2h} + \sqrt{2x}} = \frac{1}{\sqrt{2x}}$$

$$g'(x) = \frac{1}{\sqrt{2x}}$$

3. (3 points) Use our basic differentiation rules to determine each derivative.

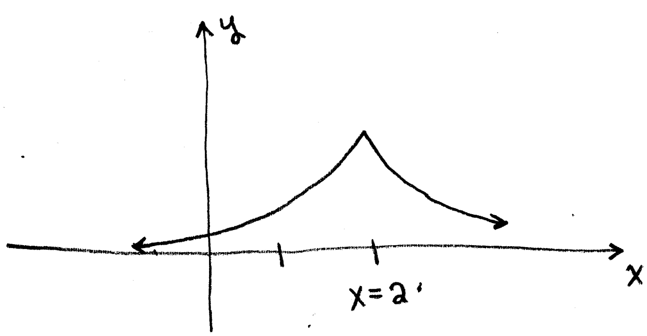
(a) $\frac{d}{dx}\left(x^{1/2} + 2x^{1/3} - 3x^{7/5}\right)$

$$= \frac{1}{2}x^{-1/2} + \frac{2}{3}x^{-2/3} - \frac{21}{5}x^{2/5}$$

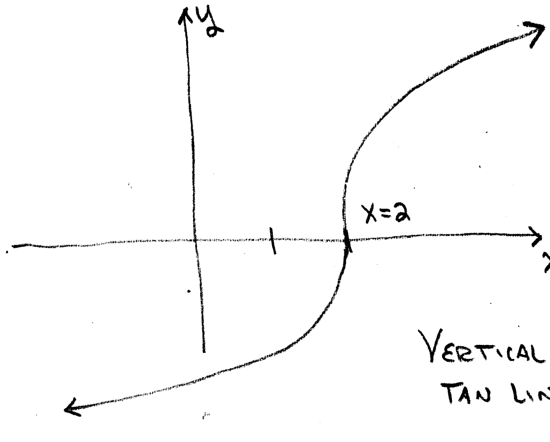
(b) $\frac{d}{dx}\left(4\cos x - 10\sin x + \frac{1}{x}\right) = \frac{d}{dx}\left(4\cos x - 10\sin x + x^{-1}\right)$

$$= -4\sin x - 10\cos x - x^{-2}$$

4. (1 point) Sketch the graph of a function that is continuous at $x = 2$, but not differentiable at $x = 2$.



SHARP POINT



VERTICAL TAN LINE