Math 131 - Quiz 5

February 24, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 3.

1. (3 points) Evaluate each derivative.

(a) $\frac{d}{dx}\left[\sqrt[3]{x^2}\tan x\right]$ = $\frac{d}{dx} x^{2/3} \tan x$

$= \frac{2}{3}x^{-1/3}\tan x + x^{2/3}\sec^2 x$

(b) $\frac{d}{dx}\left(\frac{x^2+3x-7}{\cos x}\right)$ = $\frac{(2x+3)\cos x + (x^2+3x-7)\sin x}{\cos^2 x}$

2. (2 points) Find an equation of the line tangent to the graph of $y = \frac{2x}{x-1}$ at the point where $x = -1$.

$\frac{dy}{dx} = \frac{2(x-1) - (2x)(1)}{(x-1)^2}$

$x = -1 \Rightarrow y = \frac{-2}{-2} = 1$

$m = \left.\frac{dy}{dx}\right|_{x=-1} = \frac{-2}{4} = -\frac{1}{2}$

TAN LINE:

$y - 1 = -\frac{1}{2}(x+1)$

or

$y = -\frac{1}{2}x + \frac{1}{2}$

Turn over.

3. (2 points) The following table gives the values of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ at selected values of x. Use the table for the following problems.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	3	-5
2	2	0	-1	-2

(a) Let $h(x) = 2f(x)g(x)$. Compute $h'(1)$.

$h'(x) = 2f'(x)g(x) + 2f(x)g'(x)$

$h'(1) = 2f'(1)g(1) + 2f(1)g'(1) = 2(-1)(3) + 2(3)(-5) = \boxed{-36}$

(b) Let $h(x) = \frac{1}{x} + \frac{f(x)}{g(x)}$. Compute $h'(2)$.

$h'(x) = -\frac{1}{x^2} + \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$h'(2) = -\frac{1}{4} + \frac{(0)(-1) - (2)(-2)}{(-1)^2} = -\frac{1}{4} + 4 = \boxed{\frac{15}{4}}$

4. (3 points) An object is thrown straight up from over the side of a 90-ft building with an initial velocity of 40 ft/sec. Assume that gravity is the only force acting on the object.

(a) Find the function $s(t)$ that gives the object's height at time t.

$\boxed{s(t) = -16t^2 + 40t + 90}$

(b) Find the object's maximum height.

$s'(t) = -32t + 40 = 0$

$\Rightarrow t = \frac{5}{4}$

$s\left(\frac{5}{4}\right) = -16\left(\frac{5}{4}\right)^2 + 40\left(\frac{5}{4}\right) + 90$

$= \boxed{115 \text{ FT}}$

(c) When does the object hit the ground?

$s(t) = 0 \Rightarrow t = \frac{-40 - \sqrt{40^2 + 4(16)(90)}}{-32} = \frac{5 + \sqrt{115}}{4} \approx \boxed{3.93 \text{ SEC}}$