Math 131 - Quiz 6(IC)

March 3, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Evaluate each derivative.

(a) $\frac{d}{dt}\sqrt[3]{5t^2+7t}$ = $\frac{d}{dt}(5t^2+7t)^{1/3}$

$= \frac{1}{3}(5t^2+7t)^{-2/3}(10t+7)$

(b) $\frac{d}{dx}[x^6\cos(x^2)]$ = $6x^5\cos(x^2) + x^6(-\sin(x^2))(2x)$

$= 6x^5\cos(x^2) - 2x^7\sin(x^2)$

2. (2 points) Find an equation of the line tangent to the graph of $y = (x^2+1)^{-3}$ at the point where $x = 1$.

$\frac{dy}{dx} = -3(x^2+1)^{-4}(2x) = \frac{-6x}{(x^2+1)^4}$

$m = \left.\frac{dy}{dx}\right|_{x=1} = \frac{-6}{2^4} = \frac{-6}{16} = \frac{-3}{8}$

POINT: $x = 1$

$y = 2^{-3} = \frac{1}{8}$

TAN LINE IS $y - \frac{1}{8} = -\frac{3}{8}(x-1)$

OR

$y = -\frac{3}{8}x + \frac{1}{2}$

Math 131 - Quiz 6(TH)

March 2, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This take-home portion of the quiz is due March 7.

1. (1 point) Find an equation of the line tangent to the graph of $y = \left(3x + \frac{1}{x}\right)^2$ at the point where $x = 1$.

Point: $x = 1 \Rightarrow y = (3+1)^2 = 16$

$(1, 16)$

Slope:

$$\frac{dy}{dx} = 2\left(3x + \frac{1}{x}\right)\left(3 - \frac{1}{x^2}\right)$$

$$m = \left.\frac{dy}{dx}\right|_{x=1} = 2(4)(2) = 16$$

Tangent line:

$$y - 16 = 16(x - 1)$$

or

$$y = 16x$$

2. (1 point) Determine $f'(x)$ if $f(x) = \tan^3(x^2 + \pi)$.

$$f(x) = \left(\tan(x^2+\pi)\right)^3$$

$$f'(x) = 3\tan^2(x^2+\pi)\,\frac{d}{dx}\tan(x^2+\pi)$$

$$= 3\tan^2(x^2+\pi)\sec^2(x^2+\pi)(2x)$$

$$= 6x\tan^2(x^2+\pi)\sec^2(x^2+\pi)$$

Turn over.

3. (1 point) Determine $\frac{d^2y}{dx^2}$ if $y = \cos(x^2)$.

$$\frac{dy}{dx} = -\sin(x^2)(2x) = -2x\sin(x^2)$$

$$\frac{d^2y}{dx^2} = -2\sin(x^2) + (-2x)\cos(x^2)(2x)$$

$$= \boxed{-2\sin(x^2) - 4x^2\cos(x^2)}$$

4. (2 points) Find an equation of the line tangent to the graph of the equation

$$xy^2 + \sin(\pi y) - 2x^2 = 10$$

at the point $(2, -3)$.

$$\frac{d}{dx}\left(xy^2 + \sin(\pi y) - 2x^2\right) = \frac{d}{dx}(10)$$

$$y^2 + 2xy\frac{dy}{dx} + \cos(\pi y)\left(\pi\frac{dy}{dx}\right) - 4x = 0$$

$$\left[2xy + \pi\cos(\pi y)\right]\frac{dy}{dx} = 4x - y^2$$

$$\frac{dy}{dx} = \frac{4x - y^2}{2xy + \pi\cos(\pi y)}$$

$$\left.\frac{dy}{dx}\right|_{(2,-3)} = \frac{4(2) - (-3)^2}{2(2)(-3) + \pi\cos(-3\pi)} = \frac{-1}{-12 - \pi} = \frac{1}{\pi + 12}$$

TAN LINE:

$$y + 3 = \frac{1}{\pi + 12}(x - 2)$$