

Math 131 - Quiz 7

March 24, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 31.

1. (2 points) Let $f(x) = x^3 + 4x + 3$. Compute $(f^{-1})'(3)$.

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(0)} = \boxed{\frac{1}{4}}$$

$$f'(x) = 3x^2 + 4$$

$$f^{-1}(3) = \omega \Leftrightarrow f(\omega) = 3$$

$$\omega^3 + 4\omega + 3 = 3 \Rightarrow \omega = 0 \Rightarrow f^{-1}(3) = 0$$

2. (3 points) Let $g(x) = x + \sqrt{x}$. Compute $(g^{-1})'(4)$. You will need to use your calculator (or the quadratic formula) to compute $g^{-1}(4)$.

$$(g^{-1})'(4) = \frac{1}{g'(g^{-1}(4))} \approx \frac{1}{g'(2.438447)} \approx \frac{1}{1.320194} \approx \boxed{0.75746}$$

$$g'(x) = 1 + \frac{1}{2}x^{-1/2}$$

$$g^{-1}(4) = \omega \Leftrightarrow g(\omega) = 4$$

$$\omega + \sqrt{\omega} = 4$$

⇓

$$\omega \approx 2.438447...$$

} CALCULATOR

EXACT VALUE FOR

$$\omega = g^{-1}(4) \text{ is } \frac{9 - \sqrt{17}}{2}$$

Turn over.

3. (3 points) Determine each derivative.

$$(a) \frac{d}{dx} \cot^{-1}(3x+1) = \frac{-1}{(3x+1)^2+1} \cdot 3 = \boxed{\frac{-3}{(3x+1)^2+1}}$$

$$(b) \frac{d}{dt}(te^{t^2}) = e^{t^2} + te^{t^2}(2t) = e^{t^2} + 2t^2e^{t^2} = \boxed{e^{t^2}(1+2t^2)}$$

$$(c) \frac{d}{dx} \ln\left(\frac{x^2(x+1)}{5x+4}\right) = \frac{d}{dx} \left[2\ln x + \ln(x+1) - \ln(5x+4) \right] = \boxed{\frac{2}{x} + \frac{1}{x+1} - \frac{5}{5x+4}}$$

4. (2 points) Find an equation of the line tangent to the graph of $y = \cos^{-1}(2x)$ at the point where $x = 1/4$.

$$\text{Slope: } \frac{dy}{dx} = \frac{-1}{\sqrt{1-(2x)^2}} \cdot 2$$

$$\text{Point: } x = \frac{1}{4} \Rightarrow y = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-4x^2}}$$

$$m = \left. \frac{dy}{dx} \right|_{x=\frac{1}{4}} = \frac{-2}{\sqrt{\frac{3}{4}}} = \frac{-4}{\sqrt{3}}$$

TANGENT LINE:

$$y - \frac{\pi}{3} = -\frac{4}{\sqrt{3}} \left(x - \frac{1}{4}\right)$$