

Math 131 - Quiz 8

March 31, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 7.

1. (2 points) Use logarithmic differentiation to find the derivative of $y = \frac{x^{3/2}(7x+1)}{e^x \cos^2 x}$.

$$y = \frac{x^{3/2}(7x+1)}{e^x (\cos x)^2} \Rightarrow \ln y = \frac{3}{2} \ln x + \ln(7x+1) - \ln e^x - 2 \ln \cos x$$

$$\ln y = \frac{3}{2} \ln x + \ln(7x+1) - x - 2 \ln \cos x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{2x} + \frac{7}{7x+1} - 1 + \frac{2 \sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{x^{3/2}(7x+1)}{e^x \cos^2 x} \left(\frac{3}{2x} + \frac{7}{7x+1} - 1 + 2 \tan x \right)$$

2. (2 points) Let $g(x) = \log_5(2x^3+7)^4$. Compute $g'(1)$. Write your final answer in decimal form, rounded to the nearest thousandth.

$$g(x) = 4 \log_5(2x^3+7) = \frac{4 \ln(2x^3+7)}{\ln 5}$$

$$g'(x) = \frac{4}{\ln 5} \frac{6x^2}{2x^3+7}$$

$$g'(1) = \frac{4}{\ln 5} \left(\frac{6}{9} \right) = \frac{8}{3 \ln 5} \approx \boxed{1.657}$$

Turn over.

3. (2 points) Let $b(x) = 2^{4x} + 4x^2$. Determine $b'(x)$.

$$b'(x) = (2^{4x})(4 \ln 2) + 8x$$

4. (2 points) Find the linearization of $f(x) = x^2 + x^{1/2} + \frac{1}{x}$ at $x = 1$, then use it to approximate $f(0.98)$.

$$f(1) = 3.$$

$$f'(x) = 2x + \frac{1}{2}x^{-1/2} - \frac{1}{x^2}$$

$$f'(1) = 2 + \frac{1}{2} - 1 = \frac{3}{2}$$

$$L(x) = 3 + \frac{3}{2}(x-1)$$

$$f(0.98) \approx L(0.98)$$

$$= 3 + \frac{3}{2}(-0.02)$$

$$= 2.97$$

5. (2 points) Use differentials to approximate the change in $y = \sqrt{x^3 + 1}$ as x changes from 2 to 2.07.

$$\Delta y \approx \frac{dy}{dx} \Delta x$$

$$\frac{dy}{dx} = \frac{1}{2}(x^3 + 1)^{-1/2}(3x^2)$$

$$\left. \frac{dy}{dx} \right|_{x=2} = \frac{1}{2}(9)^{-1/2}(12) = 2$$

$$\Delta x = 0.07$$

$$\Delta y \approx (2)(0.07) = 0.14$$