

Math 131 - Quiz 9

April 6, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 11.

1. (2 points) Find the linearization of $f(x) = \sqrt[3]{x}$ at $x = 27$. Then use your linearization to approximate $\sqrt[3]{26}$.

$$f'(x) = \frac{1}{3} x^{-2/3}$$

$$f'(27) = \frac{1}{27}$$

$$f(27) = 3$$

$$L(x) = 3 + \frac{1}{27}(x - 27)$$

$$\begin{aligned} \sqrt[3]{26} &\approx L(26) = 3 + \frac{1}{27}(26 - 27) \\ &= 3 - \frac{1}{27} \\ &\approx \boxed{2.963} \end{aligned}$$

2. (2 points) Use a linearization to approximate $(0.99)^8$.

$$f(x) = x^8$$

$$f'(x) = 8x^7$$

Will use the
linearization at $x=1$

$$L(x) = f(1) + f'(1)(x-1)$$

$$L(x) = 1 + 8(x-1)$$

$$(0.99)^8 \approx 1 + 8(0.99 - 1) = \boxed{0.92}$$

3. (2 points) The surface area of a sphere is given by $S = 4\pi r^2$, where r is the radius of the sphere. Use differentials to find the approximate percent change in surface area if the percent change in radius is 0.5%.

$$\frac{dS}{dr} = 8\pi r$$

$$\Delta S \approx 8\pi r \Delta r$$

$$\Delta r = 0.005r$$

$$\Delta S \approx 8\pi r (0.005r) = 0.04\pi r^2$$

$$\begin{aligned} \frac{\Delta S}{S} &= \frac{0.04\pi r^2}{4\pi r^2} = 0.01 \\ &= \boxed{1\%} \end{aligned}$$

Turn over.

4. (2 points) Use calculus techniques to find the absolute extreme values of $f(x) = 3x^{2/3} - 2x$ on $[-1, 1]$.

$$f'(x) = 2x^{-1/3} - 2 = \frac{2}{\sqrt[3]{x}} - 2$$

$$f'(x) = 0 \Rightarrow x^{-1/3} = 1$$

$$\Rightarrow x = 1 \text{ (END PT)}$$

$$f'(x) \text{ DNE} \Rightarrow x = 0 \text{ (CRIT \#)}$$

$$\text{CRIT \# : } x = 0$$

$$\text{ENDPTS : } x = -1$$

$$x = 1$$

x	f(x)
0	0 ← ABS MIN
-1	5 ← ABS MAX
1	1

5. (2 points) Use calculus techniques to find the absolute extreme values of $g(x) = \frac{x}{x+3}$ on $[-1, 6]$.

$$g'(x) = \frac{(x+3)(1) - (x)(1)}{(x+3)^2} = \frac{3}{(x+3)^2}$$

THERE ARE NO CRIT. #'S

x	g(x)
-1	$-\frac{1}{2}$ ← ABS MIN
6	$\frac{2}{3}$ ← ABS MAX